A Cascade-Based Emergency Model for Water Distribution Network

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Water distribution network

- Urban water supply system can be construed as a complex network with various physical edges and the performance of water distribution network (WDN) can be studied by the complex network theory.

- The robustness of WDN refers to the ability to avoid the loss of functions and the ability to tolerate errors and failures after the whole network damage or the components damage.

- WDN is a geography-related distributed system and in correlation with geographic space, WDN is highly sensitive to natural disasters (e.g., earthquake, hurricane) or human attack (e.g., terrorist attack). In WDN, the failures of the critical components arising from the destruction may lead to people and property loss and even impact the economic and social development.

Cascading failures in WDN

- Cascading failure, as a step-by-step failure process, is a topic of recent interest on robustness and network security.

- A large amount of loads exist in WDN. The failure of a certain component (such as breakdown or attack) may redistribute the network load, which may further lead to the result that certain components suffer a failure caused by the exceeding of their bearing capacity. The failures of these components may be likely to result in a secondary failure and a chain reaction. After that, a large number of components and even the whole network may collapse, which cause severe damage to the network.
- In reality, there always exist emergency response mechanisms to reduce the loss of failures and restore the normal service function. Upon the failures of WDN, external emergency forces can intervene in the failing components to cope with emergencies and assist in their restoration.

- The paper studies the emergency recovery strategies on WDN with the approach of complex network and cascading failures. The model of cascade-based emergency for WDN is built which considers the network topology analysis and hydraulic analysis to provide amore realistic result.

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The Cascade-Based Emergency Model of WDN

The Topology Structure:

Characterized by the network-like topological structure, WDN can be analyzed by the graph theory. Since the water in pipe flows along a certain direction, WDN is a directed graph. Water reservoirs, consumers, and tanks can be abstracted as nodes. Pipes, pumps, and valves can be represented as edges. The neighboring nodes are connected by pipes.

The incidence matrix N is used to describe the relationship between the nodes and pipes in the network. The number of rows is equivalent to the nodes, and the number of columns is equivalent to the pipes, respectively. N_{ij} in the matrix can be expressed as

$$N_{ij} = \begin{cases} 1, & \text{Node } i \text{ is the initial point of pipe } j \\ -1, & \text{Node } i \text{ is the terminal point of pipe } j \\ 0, & \text{Node } i \text{ is the unconnected with pipe } j \end{cases}$$

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Parameters

- ***Load:***

Load exists on the components such as nodes and pipes in WDN. Both the excessively large and small loads result in the flow changes, which may further trigger a series of cascading failures. After attack, the WDN redistributes the water pressure and flow in the network according to the topological structure and hydraulic changes. The complex network distributes the network load according to betweenness.

To study the cascade propagation of WDN, the node service pressure P^{ser} in the normal operation is used as the initial load.

Parameters

- ***Capacity:***

Capacity is the load that a node can bear. If the load exceeds its bearing capacity, components suffer from failures. These failures cause the flow redistribution, leading to secondary failures. With the expansion in urban size, WDN has been further expanded based on the original construction size. Water demand changes randomly with the requirements of urban population and industry. As a result, the node pressures are no longer the initial value but are left in a changing state. For WDN that keeps running for a long time, there is a need to take account of the changes in node pressure and measure the robustness of WDN by its new constraints.

- The node minimum capacity is defined as the acceptable minimum water pressure ${P^{min}}_k$.
- The node maximum capacity is defined as follows: $P_k^{\max} = (1 + \alpha)\, P_k^{\mathrm{ser}}$

  where ${P^{ser}}_k$ is the service pressure of node k. The service pressure obtained in normal operation ensures that all the water demand can be satisfied. The node maximum capacity refers to the upper limit of the nodes constrained by cost or aging. $\alpha \geq 0$ expresses the node tolerance, which implements the control over the intensity of initial load and load distribution - the extra pressure that a node in WDN can bear. The greater the value of α is, the bigger the difference between the components will be (load distribution will be more non-uniform).

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Parameters

The network remains stable in the initial stage. The load of every node is smaller than its capacity. After the failure occurs, the load on the failure node is distributed to its neighboring nodes. If the neighboring nodes are unable to process the extra load (i.e., the load exceeds its capacity), the neighboring nodes fail and then result in cascading failures. If the redistributed node pressure is within its capacity, the node can still supply water; otherwise, the node is recognized as a new avalanched node. The connected pipes are closed to avoid risk expansion. Failure propagation process:

$$\text{New avalanched node} = \begin{cases} 0, & P_k^{\min} < P_k < P_k^{\max} \\ 1, & \text{otherwise.} \end{cases}$$

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Parameters

- *Actual Demand*

The demand supplied equals the required one as a customer controls a faucet when the system capacity is not exceeded. The demand becomes the maximum allowed by the actual pressure in pressure deficient condition.

In addition, the node pressure should be neither too low nor too high. Abnormally high pressures may cause aging pipes to burst and lose service functions. Therefore, each node pressure must be controlled between the maximum pressure and the minimum pressure. When $P^{\max}_{k} \leq P_{k}$, the abnormally high-pressure condition occurs. To avoid risk expansion, it is assumed that the section where the failure demand node covered is isolated from the rest of the network. The failure node is removed out of the network and its connected pipes are closed.

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Parameters

The relationship between actual demand and node pressure is revealed in combination with the minimum and maximum pressures.

$$Q_{k,t}^{\mathrm{act}} = \begin{cases} 0 & P_{k,t} \le P_k^{\mathrm{min}}, \\ Q_{k,t}^{\mathrm{req}} \sqrt{\dfrac{P_{k,t} - P_k^{\mathrm{min}}}{P_k^{\mathrm{ser}} - P_k^{\mathrm{min}}}} & P_k^{\mathrm{min}} < P_{k,t} < P_k^{\mathrm{ser}}, \\ Q_{k,t}^{\mathrm{req}} & P_k^{\mathrm{ser}} \le P_{k,t} < P_k^{\mathrm{max}}, \\ 0 & P_k^{\mathrm{max}} \le P_{k,t}, \end{cases}$$

where $Q^{act}{}_{k,t}$ is the actual demand of node k at time t. $Q^{req}{}_{k,t}$ is the required demand (full demand) of node k at time t. It can be obtained when WDN performs normally. $P_{k,t}$ is the pressure of node k calculated at time t. $P^{min}{}_k$ is the minimum pressure of node k. $P^{ser}{}_k$ is the service pressure of node k. $P^{max}{}_k$ is the maximum pressure of node k.

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The Cascading Failure Model

- *EPANET 2*

EPANET 2 is open source software developed by the United States Environmental Protection Agency (EPA). EPANET 2 can simulate the water hydraulic and water quality of WDN in a certain period of time.

- *Attack Pattern*

Cascading failures can be triggered by random attack and intentional attack. The target of intentional attack is a network component with some special feature, for example, the most connected node, the highest betweenness node, or the lowest betweenness node. Attack on the special component can cause a more rapid network failure. These special components can be regard as vulnerability components. When the network information is entirely unknown, the components in the network can only be attacked randomly. However, in actual network, the vulnerability differs from component to component. When the network information can be completely or partially obtained, the network can be break down rapidly if priority is given to the vulnerable ones.

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The Cascading Failure Model

From the perspective of infrastructure protection, the intentional attack is chosen. The actual network shows high tolerance for random failures. The attack target is constituted by the node-based attacks. The failure of the nodes or pipes in WDN causes changes to the topological structure and water properties. It is necessary to update the WDN model before the hydraulic analysis. The update of WDN involves two aspects:

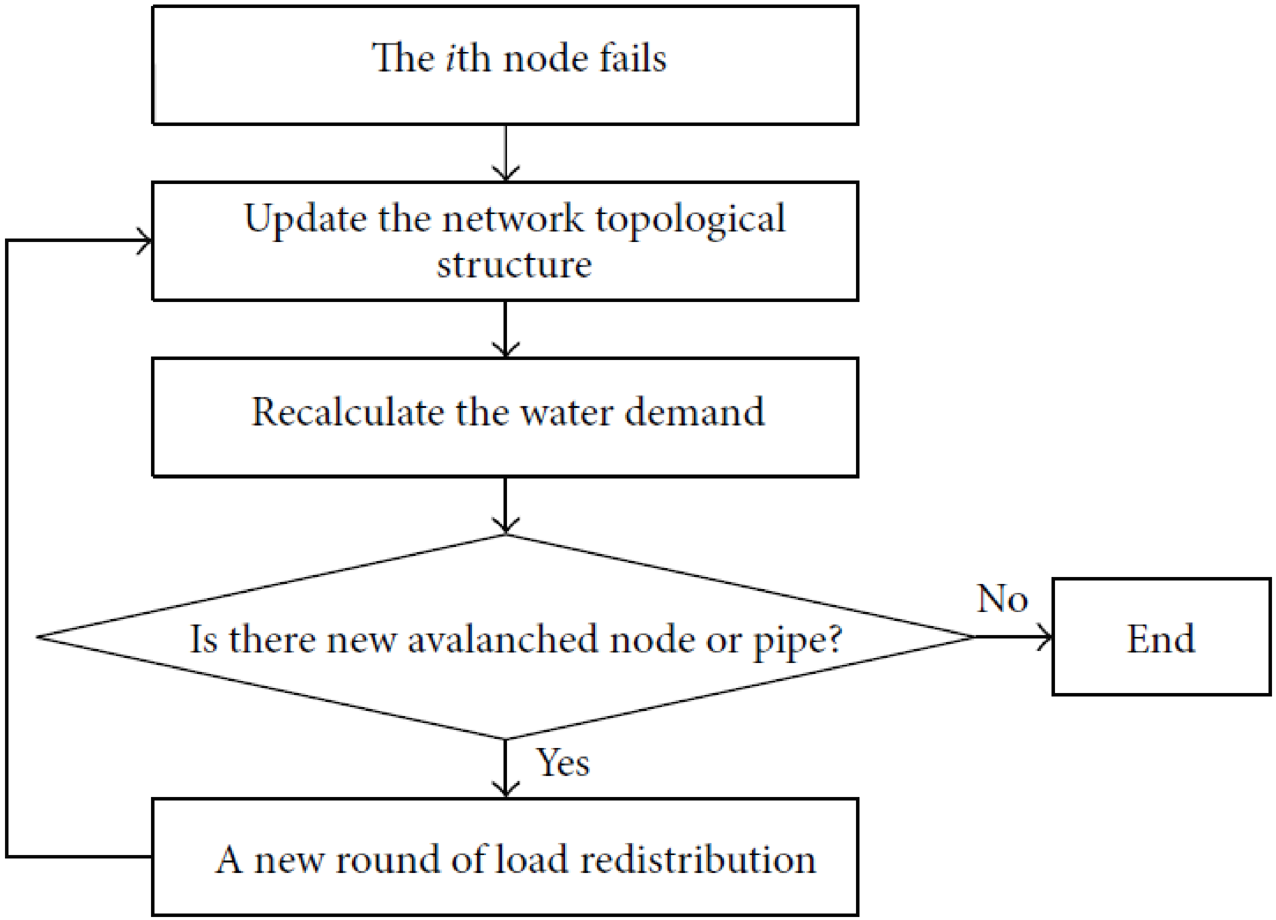
(1) the update of the topological structure, such as shutting down the pipes within the influencing scope of accidents and

(2) recalculating the water demand within the influencing scope.

The influence of node failure in WDN can be described as follows:

- After node failure, the node will be isolated to prevent loss - the failure node is removed out of WDN.
- In EPANET 2, the actual demand is set as zero and the upstream and downstream pipes associated with the failure node are shut down.
- The stop of cascading failures of the WDN is defined as no new failure nodes or pipes appeared, that is, the network returns to stable state again.

Flowchart of the node-based attack in WDN



The Emergency Model

- *The Resource Threshold Value*

New failure nodes appear after the intentional attack. The failure nodes may trigger the hydraulic redistribution of WDN. In real life, emergency response can help people to reduce losses. After a natural disaster, manpower with technical support, vehicles, and emergency supplies are involved in recovery activities to fight the disaster. These manpower, vehicles, and emergency supplies can be regarded as *external emergency resources*.

The Emergency Model

The threshold value of the emergency resources is defined. The threshold value of the low-pressure resources $P^{res}_{min,k}$ is expressed as

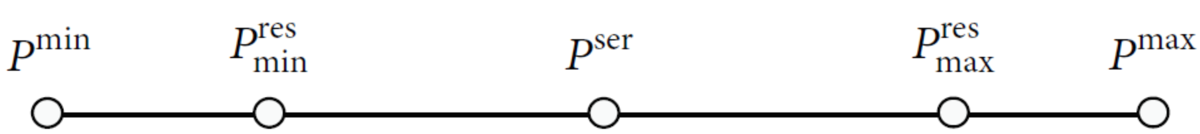
$$P^{\mathrm{res}}_{\mathrm{min},k} = P^{\mathrm{min}}_{k} \times (1 + \beta)$$

The threshold value of the high-pressure resources $P^{res}_{max,k}$ is expressed as

$$P^{\mathrm{res}}_{\mathrm{max},k} = P^{\mathrm{max}}_{k} \times (1 - \beta)$$

*β is the parameter of external emergency resources

The original water pressure in WDN is expanded:



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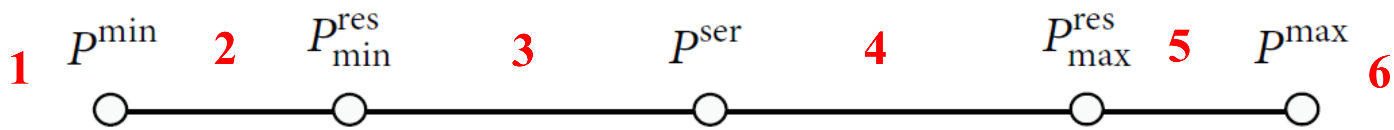


(1) When the node pressure is smaller than the minimum pressure, the excessively low water pressure leads to flow interruption and short supply. The node actual demand becomes zero. External emergency resources can be introduced for restoration.

(2) When the node water pressure is greater than the minimum pressure but smaller than the low-pressure resource threshold, the node suffers low-pressure challenge. If the external resources are not introduced, the water pressure will keep declining and then lead to flow interruption. In this situation, external emergency resources can be introduced to restore the nodes within the interval of the water pressures mentioned above.

(3) When the node water pressure is higher than the low-pressure resource threshold but lower than the service pressure, the water is supplied in a reduction level. The actual demand can be calculated with (4).

(4) When the node water pressure is higher than the service water pressure but lower than the high-pressure resource threshold, the node maintains normal supply. The actual demand is equal to the required demand.

(5) When the node water pressure is higher than the high-pressure resource threshold but lower than the maximum pressure, the node suffers high-pressure challenge. If the external resources are not introduced, the pipe burst may arise from the continuous increase in water pressure. In this situation, external emergency resources can be introduced to restore the nodes within the interval of the water pressures mentioned above.

(6) When the node water pressure is higher than the maximum pressure, it easily results in pipe burst and then cascading failures. Under this circumstance, the node fails and the actual flow is zero. In view of this, the node can be restored by the external emergency resources.

The Emergency Model

- *The Load Redistribution*

The water pressures of every node can be obtained by the recalculation of network hydraulic under failure conditions. Judge the interval of water pressure where the node pressure exists. Suppose the water pressure of node k is $P_{k,t}$ at time t. The external emergency resources should be distributed to the nodes with the recovery strategies if $P_{k,t}$ is greater than the maximum resource threshold or smaller than the minimum resource threshold. After restoration, the final load is $P'_{k,t}$. The load redistribution process can be expressed as follows:

$$P'_{k,t} = \begin{cases} P_{k,t} \times (1 + w_{k,t} \times R) & P_{k,t} \leq P^{\text{res}}_{k,\min}, \\ P_{k,t} & P^{\text{res}}_{k,\min} < P_{k,t} < P^{\text{res}}_{k,\max}, \\ P_{k,t} \times (1 - w_{k,t} \times R) & P_{k,t} \geq P^{\text{res}}_{k,\max}, \end{cases}$$

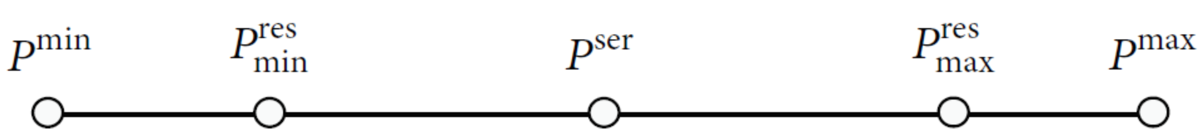
where $w_{k,t}$ represents the weight for node k at time t. The weight describes the resources that are introduced according to different recovery strategies. R is the total available resources.

The Emergency Model

After the node is allocated with a certain quantity of emergency resources, the final load $P'_{k,t}$ suggests that node k only has three states:

- normal state ($P^{res}_{k,min} < P_{k,t} < P^{res}_{k,max}$)
- challenge state ($P_{k,min} < P_{k,t} \le P^{res}_{k,min}$ or $P^{res}_{k,max} < P_{k,t} \le P_{k,max}$)
- avalanched state ($P_{k,t} \le P_{k,min}$ or $P_{k,t} \ge P_{k,max}$)

The node with the challenge state can still ensure water supply. However, if the node suffers avalanche, the node fails and then it is removed out of the WDN.



Node Betweennes

Weight is devoted to formulating the recovery strategies from three aspects including uniform distribution, node betweenness, and node pressure. The recovery strategies are considered from two aspects, that is, the network topological structure and the collapse information. The node betweenness is defined as follows:

$$B_k = \sum_{a \neq b} \frac{\sigma_{ab}(k)}{\sigma_{ab}}$$

where $\sigma_{ab}(k)$ is the shortest path between node a and node b passing through node k. σ_{ab} is the sum of all the shortest paths between node a and node b. In most cascading failure studies, node betweenness is used to measure the network topological property. It evaluates the node influence from the aspect of topology. The larger the node betweenness is, the greater the influence will be.

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The Recovery Strategies

RS 1: the resources are distributed to all nodes uniformly.

RS 2: the resources are distributed to all nodes according to the node betweenness.

RS 3: the resources are uniformly distributed to the challenged nodes.

RS 4: the resources are distributed to the avalanched nodes uniformly. If there are no avalanched nodes, the resources are uniformly distributed to the challenged nodes.

RS 5: the resources are distributed to the avalanched nodes according to node betweenness. If there are no avalanched nodes, the resources are distributed to the challenged nodes according to node betweenness.

RS 6: the resources are distributed to the avalanched nodes according to the node pressure. If there are no avalanched nodes, the resources are distributed to the challenged nodes according to the node pressure.

RS 7: no emergency resource is introduced to the WDN.

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The Recovery Strategies

- *<u>The Robustness Evaluation Index</u>*

The relative size of the nodes G in the largest connected component is as follows:

$$G = \frac{N'}{N}$$

The avalanched size of nodes AS is as follows: $\mathrm{AS} = \frac{\sum \mathrm{as}_i}{N}$

The challenged size of nodes CS is as follows: $\mathrm{CS} = \frac{\sum \mathrm{cs}_i}{N}$

The cascade propagation velocity is as follows: $V = \frac{N - N'}{T}$

The Recovery Strategies

N' is the nodes in the largest connected component of WDN after the cascading failures stop, N is the number of the demand nodes in WDN, Σas_i and Σcs_i are the number of the avalanched and the challenged nodes in WDN after the stop of the cascading failures resulting from the attack against node i, and T is the total number of iterations that measure the time steps of cascade propagation in the network.

- G reflects the largest connected component after cascading failures. It can quantify the robustness of structure against cascading failures. The network has good robustness as G approaches one.
- On the contrary, AS evaluates the avalanched size. The network has poor robustness as AS approaches one.
- CS describes the challenged condition. If effective recovery strategy can be given to these nodes, the possibility of network back to normal operation can be improved.
- In addition, V measures the quantity of avalanched nodes in unit time. The smaller the V is, the less the collapse nodes per unit time will be.

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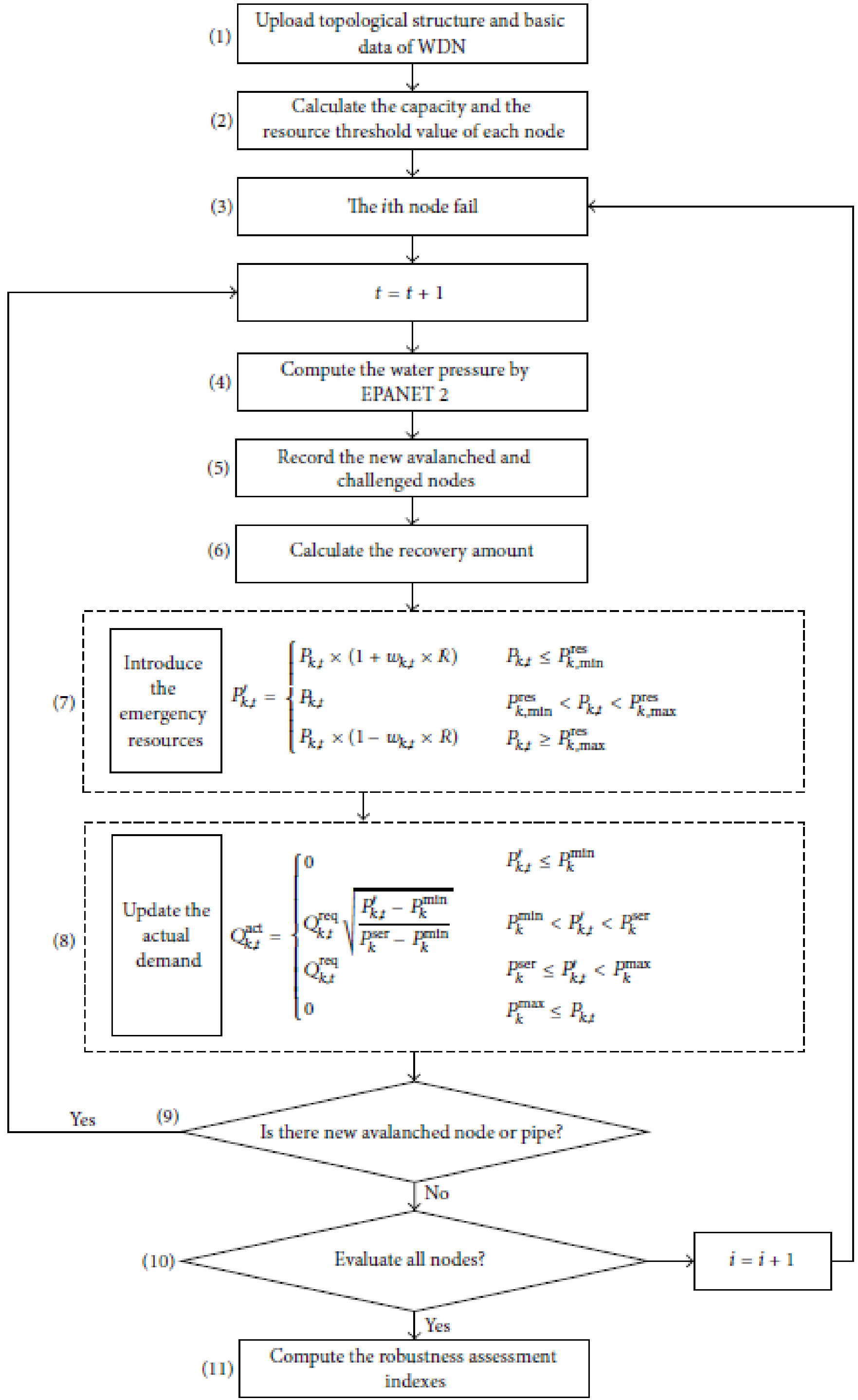
Flowchart



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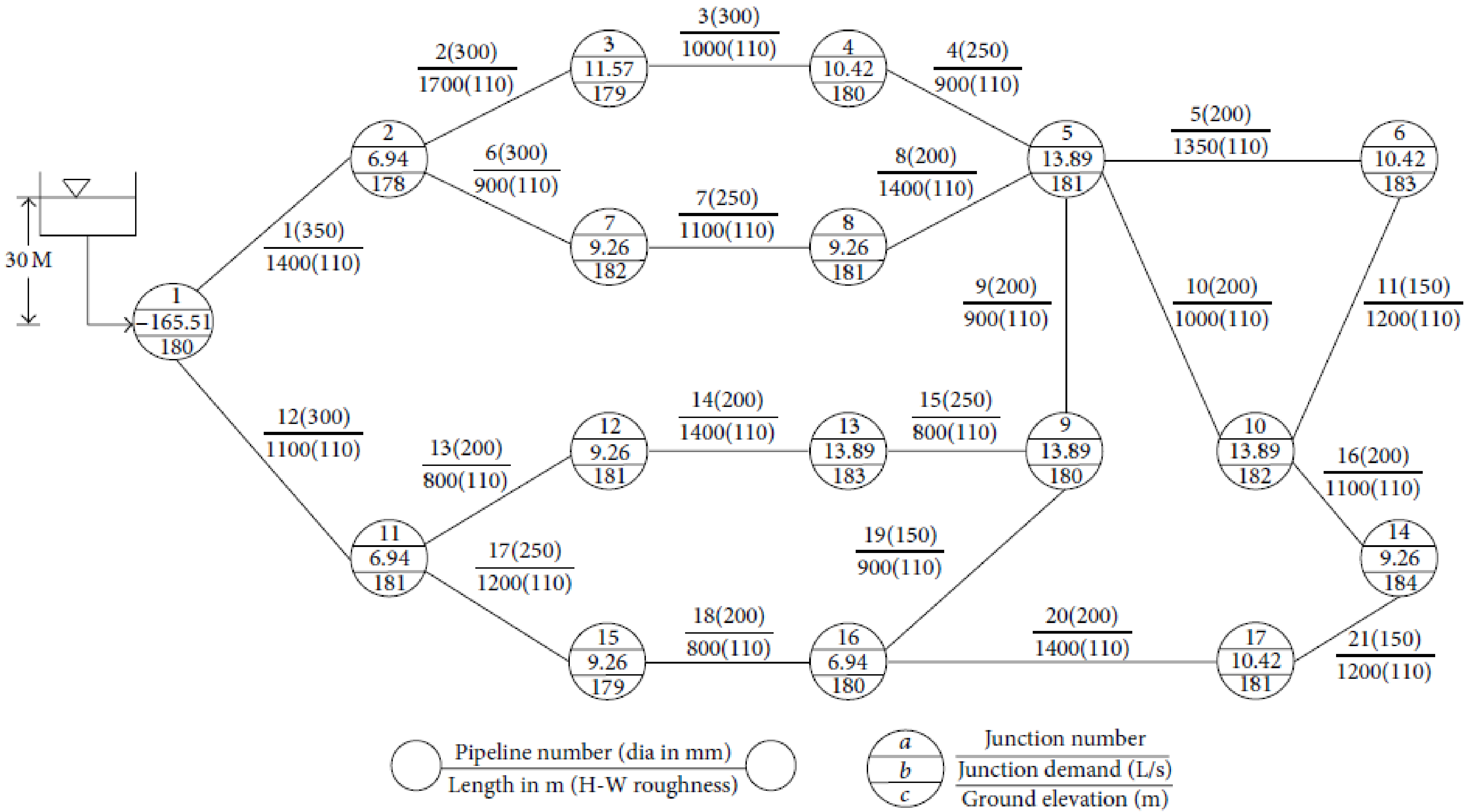
Case Study



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Case Study

- The network is represented as 17 nodes and 21 pipes. The network topological structure, node elevation, base demand, pipe diameter, length, and Hazen-William roughness are shown. The total head is 30 m. The total pipe length is 23.55 km. The pipe lengths range from 800m to 1700 m. The pipe diameters vary from150mmto 350mm.The base demands vary from 6.94 L/s to 13.89 L/s. The minimum pressure of every node is 6m.

- EPANET 2 is used to calculate the node pressure under normal condition. The node pressure is shown in Table 1.The pressure of node 14 is the smallest P_{14} = 7.12m. The pressure of node 1 is the largest P_1 = 30m. The acceptable minimum pressure of all consumer nodes is 6m. The value range of tolerance parameter α is defined as 0~1.0 to cover all possible maximum capacities. R = 0.5.

- To evaluate the recovery strategies, the cascading failure process from node 1 to node 17 is simulated successively to calculate the relative size G, the avalanched size AS, the challenged size CS, and the cascade propagation velocity V of every node until the network becomes stable again. The evaluation result with each α is the average of evaluation index from node 1 to node 17.

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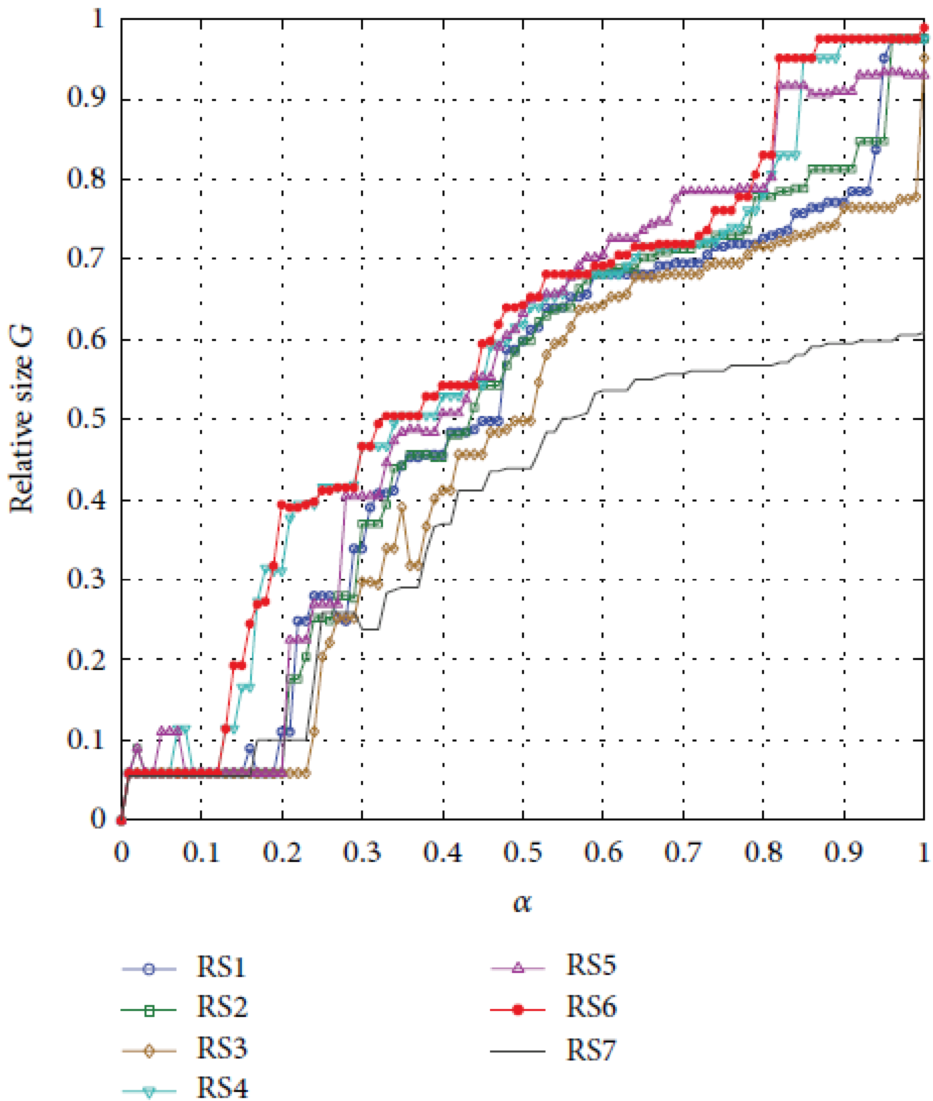
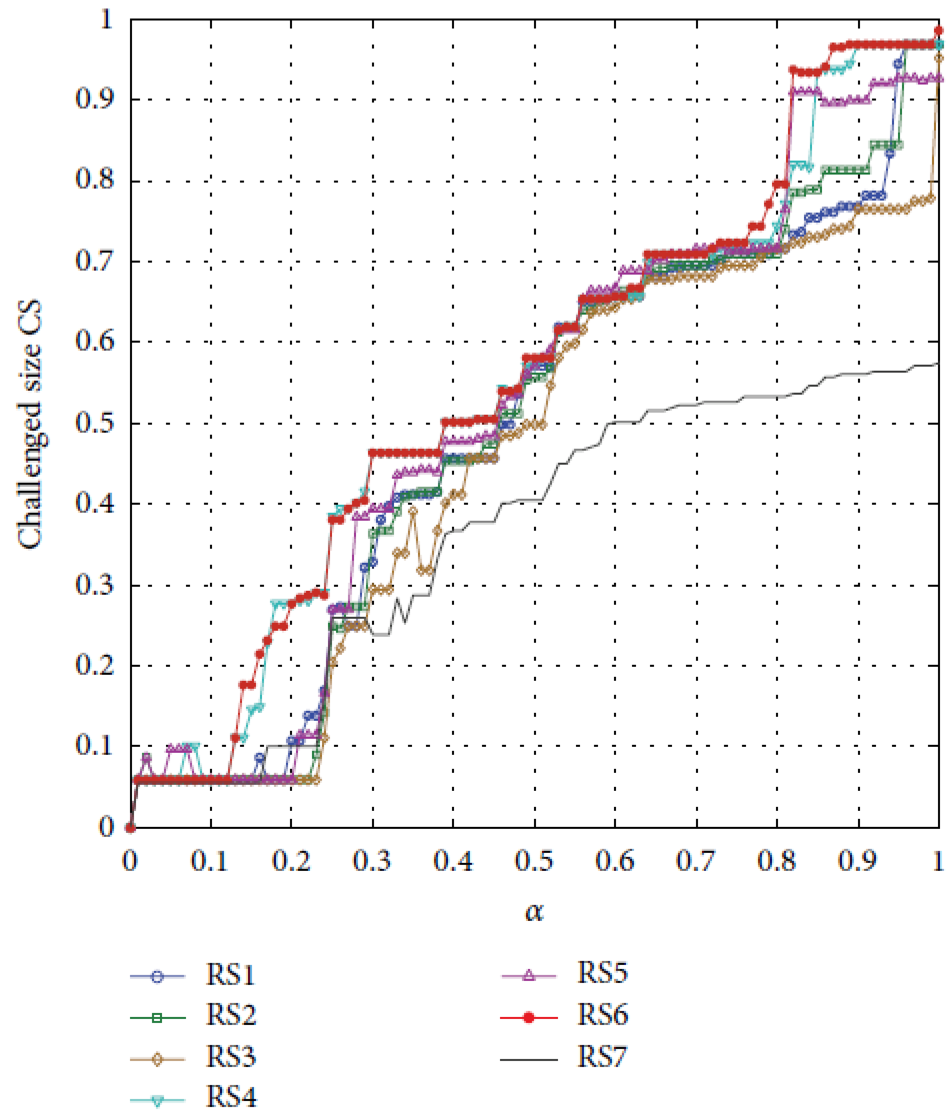
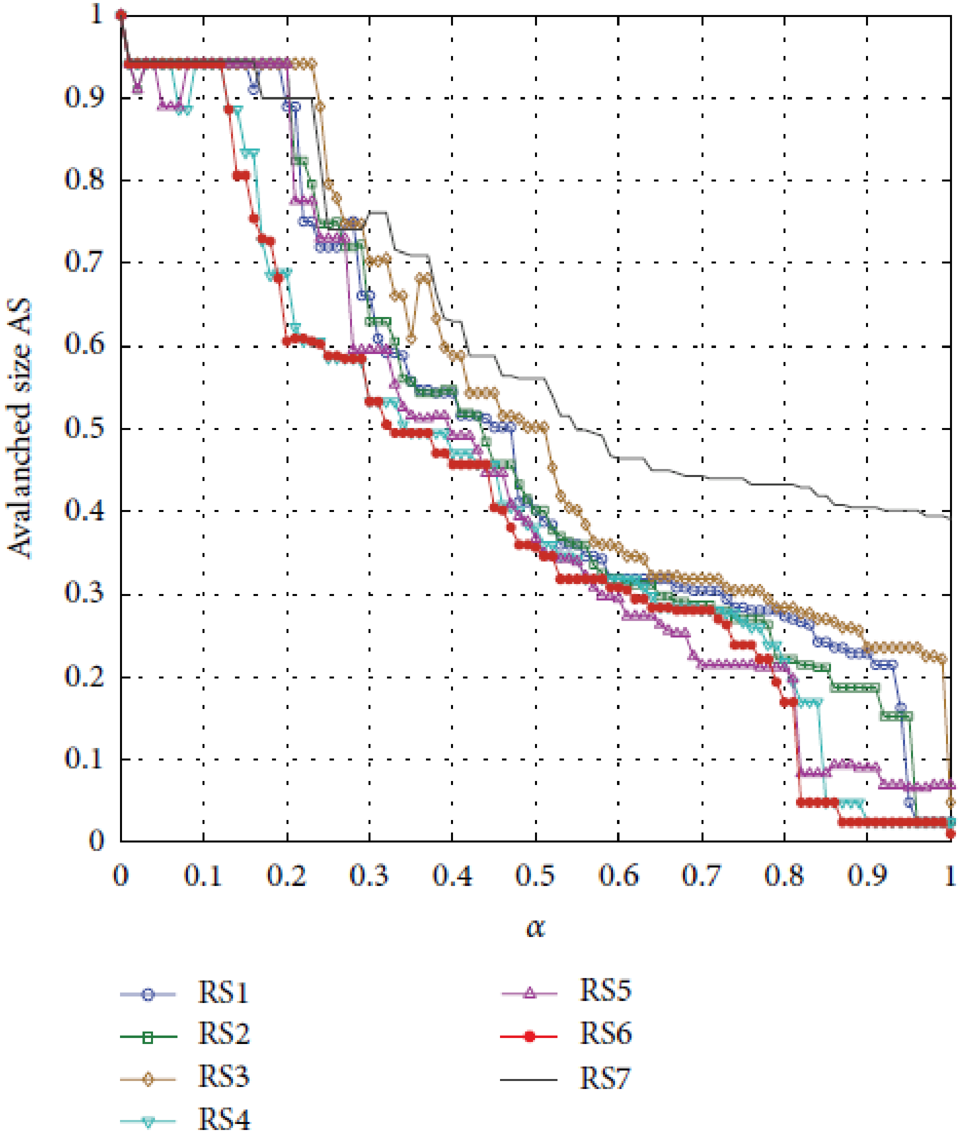
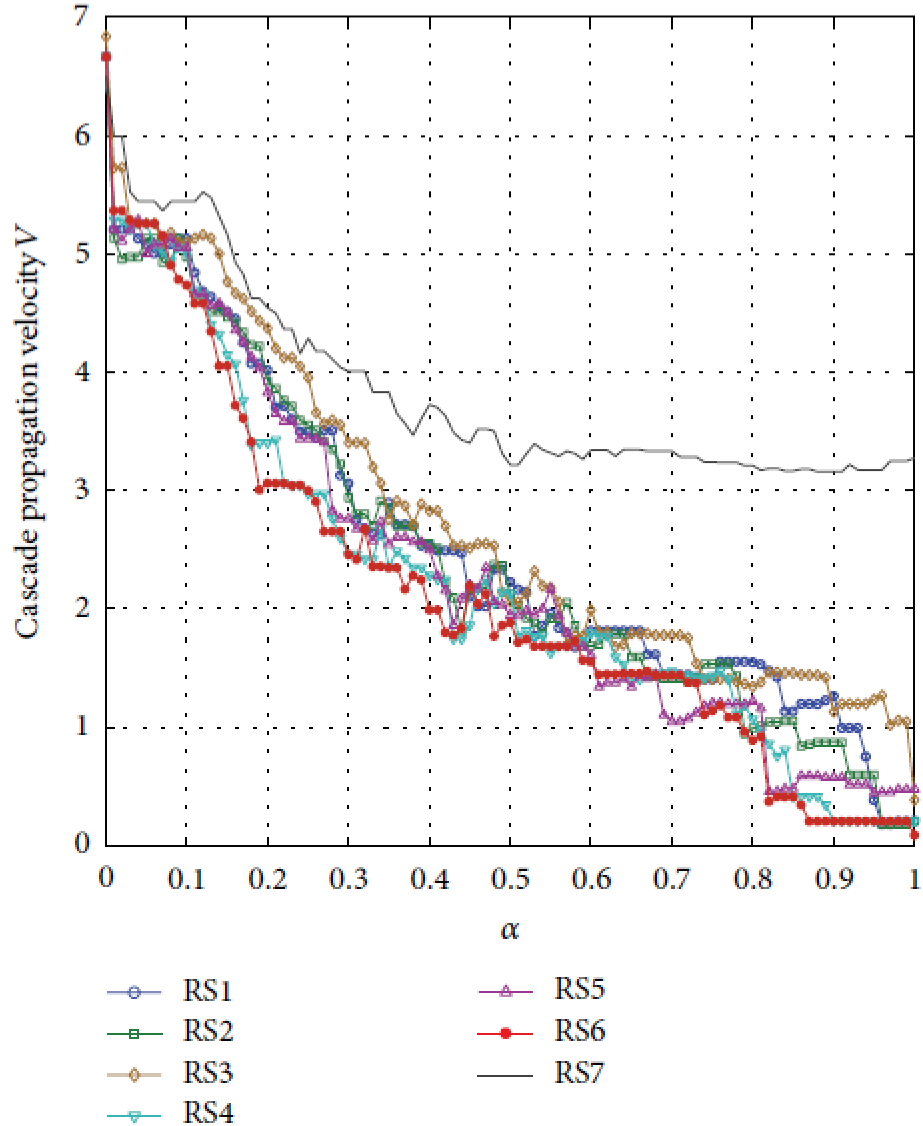


Relative size G
α
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
RS1
RS2
RS3
RS4
RS5
RS6
RS7



Challenged size CS
α
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
RS1
RS2
RS3
RS4
RS5
RS6
RS7





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Conclusion

- This paper studies the recovery strategies of urban WDN with cascading failures. The choice of the best strategy to fight the cascading failures is discussed.

- The case study manifests that the recovery strategy of node pressure is able to improve the connectivity of the WDN. Besides, this strategy can reduce the avalanched nodes and the number of avalanched nodes per time. The recovery strategy based on node pressure is superior to that based on node betweenness. For WDN, focus should be paid not only on the network topological, but also on the hydraulic property. The node pressure strategy considers both the topology structure and the balance of water supply and demand, which makes this strategy superior to others.

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