Directionality As Needed - Achieving Connectivity in Wireless Mesh Networks

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Abstract: We study how to achieve a desired connectivity in a wireless mesh network using beamforming antennas. We show that there is no unique solution to this problem. Then, we propose a simple algorithm, Directionality As Needed (DAN), which strikes a good balance between the dual goals of minimum network design cost and minimum interference among different links.

Key Words: Wireless Mesh Network, Connectivity, Beamforming, Directional Antenna.

I. INTRODUCTION

Multiple antenna elements can be used for directional beamforming. Directional antennas (DAs) focus the available power into desired directions as opposed to omni-directional transmission where the power is equally dispersed in all directions. DAs can thus provide the benefits of increased range, reduced interference, and increased spatial reuse of bandwidth. DAs have been studied for cellular networks and have been deployed for cell-sectoring. DAs have also been studied for medium access control in ad hoc networks and routing protocols. But the problem of achieving a desired connectivity in a wireless mesh network through beamforming has received little or no attention.

Our motivation stems from the idea of providing Internet connectivity using only minimal wired infrastructure to both residential users and businesses in a city like San Francisco. One solution is to deploy a wireless mesh network (WMN) with fixed wireless routers and gateways at select locations in the city. The gateways are connected to the Internet through high-speed optical fibers, and all the wireless routers have at least one path to one of the gateways. This solution is very attractive to service providers who want to get excellent market share with low-cost infrastructure (for installation and maintenance). Based on market research, a service provider would place its routers and gateways at specific locations in the city. This naturally leads to two problems: 1) where to place the gateways? and 2) what is the amount of directional sectoring at each wireless router? The first question has been partially addressed in [5]. Here, we attempt to answer the second question.

II. CONNETIVITY BETWEEN TWO NODES

For this study we use the “Cone-plus-ball” antenna model which accurately captures the essentials of a DA without the complexity of replicating an exact antenna radiation pattern. We use a relation between the beam-width $\theta_m$, main-lobe gain $g_m$, and side-lobe gain $g_s$, which we state below without proof for lack of space (please see [7] for details):

$$g_m(\theta_m, g_s) = \eta \Delta - g_s(\Delta - 1)$$

where $\Delta = 2/(1 - \cos(\theta_m/2))$ is the directivity of the antenna. For a given side-lobe gain $g_s$ we can calculate $g_m$ as a function of $\theta_m$, i.e., $g_m(\theta_m)$.

For the propagation environment, we use the standard log-distance path loss model with a path-loss exponent of $n = 4$ which is most practical in an urban setting with many buildings and obstructions.

There are two kinds of beamforming strategies: beam-switching and beam-steering. In a steered-beam system, the antenna main lobe is automatically and continuously steered to the direction of the target node (and is a favorite in military applications where cost is not a major issue) while a switched-beam system is an extension of the cellular sectorization scheme. Our study assumes a switched-beam system which is attractive for wireless mesh networks due to its lower cost.
enough for the nodes to be connected. Note that $g_m$ is related to beam-width through Eqn. (1). Due to the interplay between power and beam-width, there is no unique solution to this problem, i.e., connectivity can be achieved with different combinations of power and beam-width. In Fig. 1, we plot the minimum power required to achieve connectivity versus beam-width for various link distances with $N_0 = -90$ dB, $g_s = -10$ dB, and $\beta = 10$ dB.

One extreme is to use omni-directional transmission at all nodes. This solution would require large power and would result in large amount of interference among different links in the network. The other extreme is to use narrow beams at all nodes. Although this solution is attractive from the point of view of power consumption and interference, it will have high design cost because DAs are much costly.

III. THE MESH CONNECTIVITY PROBLEM

In this problem, the node locations are given along with the connectivity that is desired. Each node is equipped with a single radio capable of beam-switching. Also given is the maximum power $P_{\text{max}}$ that can be used at any node. The problem is to determine the beam-widths for each node and the power levels and the specific beam index which is to be used for each link. In a beam-switching solution, the allowed values of beam-width, in order to get integral non-overlapping sectors, should be factors of $360^\circ$. Specifically our beam-widths are from the following set: $S = \{360, 180, 120, 90, 72, 60, 45, 40, 36, 30, 20, 10, 5\}$. Very low beam-widths are infeasible and therefore not included in $S$. Once a beam-with $\theta_m$ is chosen, the number of sectors is $K = 360/\theta_m$. We use a global reference axis with respect to which beam orientation is specified. This can be easily achieved by using a compass when installing the antennas at any node. At any node, the center of sector 0 is along the positive x-axis (towards east) and spans an angle of $\theta_m$. The $K$ sectors indexed from 0 through $K-1$ are covered by separate beams of equal width $\theta_m$ centered at $\alpha_i$, $i = 0,...,K-1$, as shown in Fig. 2.

![Switched beamforming.](image)

Fig. 2. Switched beamforming.

![Example network connectivity (topology) embedded with omni-directional solution.](image)

Fig. 3. Example network connectivity (topology) embedded with omni-directional solution. Besides the node index, beam-width used at that node is shown. Each link has triplets of the form (Beam index, Power used, and Number of other links with which this link interferes). Since all links are bi-directional, there are two triplets on each link: the triplet above is for the link from lower-index node to higher-index node and the triplet below is for the reverse link from the higher-index node to lower-index node.

Higher resolution colored images and more details on this work [7] can be found at http://networks.cs.ucdavis.edu/~rama/DAN.html
Fig. 4. Example network connectivity (topology) with 30° solution.

Fig. 5. DAN algorithm solution with $P_{\text{max}} = 300$ mW.
Consider the example network shown in Fig. 3. It consists of 26 nodes (mesh access points) numbered from 1 through 26 and three gateways numbered 27 though 29. Gateways are special wireless access points which are connected to the backbone high-speed network through fiber. Also shown in the figure are the connectivity that is required and the low-cost all-omni-directional solution. Besides the node index at each node, the beam-width used at that node is indicated. On each link, there are triplets indicating beam index used for that link, power used for transmission over the link, and number of links which are affected due to interference caused by transmission over this link.

Fig. 3 shows that large amounts of powers are used on links and interference caused is also high. Link 18-1 requires an infeasible power (over 10,000 mW), and it causes interference to as many as 25 other links. Fig. 4 shows the solution with $30^\circ$ beam-switching antennas employed at all the nodes. Comparing with Fig. 3, we see the reduction in power and also a significant lowering of the interfering levels. In any wireless network, there is a minimum interference floor which cannot be avoided due to the nature of shared wireless medium and the desired connectivity. Using a narrow-beam solution takes us towards this minimum interference floor. But the equipment cost involved in implementing such a solution could be large, particularly if the network contains many nodes. Our objective is to find a solution which balances the design cost and interference requirements.

IV. DAN CONNECTIVITY ALGORITHM

The basic philosophy of our algorithm, Directionality As Needed (DAN), is to use only as much directionality as needed. The following two steps determine the beam-width at each node, the specific beam to be used, and the power required for each link in the network.

**Step 1: Determine the beam-width that is to be used.**

In determining beam-width, the following considerations are made. First the beam-width should be small enough so that, for all links originating from this node, the power to be used is less than the maximum. For this, we find the maximum angle $\theta_{\text{max}} \in S$ which satisfies the SNR constraint in Eqn. (2) for the longest link originating from the current node.

Second, the beam-width should be small enough so that each link originating from the given node is served on a different beam. This constraint would avoid the problem of a node’s transmission causing its neighbor to refrain from communicating with other nodes. Among all beam-widths $\theta \in S$ and for which $\theta \leq \theta_{\text{max}}$, we select the maximum which guarantees separate beams to all neighbors.

**Step 2: Based on beam-width to be used, determine power level and specific beam to be used for each link.**

When node $i$ wants to transmit to its neighboring node $j$, it determines the relative angle of node $j$, $\Phi_{ij}$, with respect to itself. Then the beam that is used by node $i$ for transmission to node $j$ is determined as:

$$\psi_j = \arg\min_{\alpha_k} \left| \Phi_{ij} - \alpha_k \right| \quad k = 0...K-1$$

The minimum power which satisfies the SNR constraint in Eqn. (2) is used for transmission over that link.

Pseudo-code for Step 1 is given in the Appendix. Step 2 is straightforward and needs no further elaboration.

V. ILLUSTRATIVE EXAMPLES

Figure 5 shows our algorithm’s solution with $P_{\text{max}} = 300$ mW. Note the different beam-widths used at the nodes. Nodes which have few neighbors and which are not far apart use large beam-widths. But nodes which have more neighbors or neighbors which are far apart use focused beams. Also note the different power levels used on different links. Power used on a link is the minimum required to support the link. Power control can reduce the amount of interference in the network.

We evaluate our algorithm by the metrics of interference and design cost. Figure 6 plots a link’s average number of interfering links as a function of $P_{\text{max}}$ allowed at each node. Note that the omni-directional solution experiences maximum interference and the $5^\circ$ narrow-beam solution experiences least interference. DAN algorithm suffers from a slightly higher interference than the narrow-beam solution. Also, for DAN, the interference initially increases with increase in $P_{\text{max}}$ but then saturates at a flat value.

Figure 7 shows the design cost for different solutions. We assume the cost of an omni-directional antenna is one unit and the cost of a beam-switching antenna of $K$ sectors is $K$ units. We see that the omni-directional solution is the cheapest and narrow-beam solutions are costly. DAN results in a costlier solution than the all-omni solution but is much cheaper than the narrow-beam solutions. Also the gap between DAN and omni-directional solutions diminishes with increase in $P_{\text{max}}$. 

![Fig. 6. Interference comparison.](image-url)
Table 1 compares the minimum, average and maximum power consumed in the network by the omni-directional solution, \(10^0\) solution, \(30^0\) solution, DAN with \(P_{max} = 300\) mW, and DAN with \(P_{max} = 1000\) mW. We observe that the minimum, average, and maximum powers consumed by omni-directional solution are large while those for the \(10^0\) beam case are small. But the amount of directionality required at each node to get a \(10^0\) beam is large and the cost involved is high. One unique aspect of DAN is that we can design a network based on the maximum power allowed. The power consumed when \(P_{max} = 300\) mW is less than that consumed when 1000 mW is used, but a 1000 mW solution requires less directionality and consequently less cost. DAN provides minimum interference at low costs. Thus, DAN provides a pragmatic way of designing WMNs depending on the requirements and the amount of resources available.

REFERENCES


APPENDIX

Pseudocode for DAN

:\textbf{Input:} G(V, E) is graph representing the desired connectivity. V is the set of nodes with their respective locations. E is the set of links that are desired.

:\textbf{Output:} \(\theta_m\) for all \(u \in V\)

1. for all \(e = (i, j) \in E, w(e) = dist(i, j)\)
2. \(S = \text{Set of all allowed beam-widths}\)
3. for each node \(u\)
4. \(V_u = \text{set of neighbors of } u\)
5. Find node \(v = \text{farthest neighbor from } u\)
6. \(d_{max} = w(u, v)\)
7. \(\theta_{max} = \text{Max } \theta \in S \text{ such that } \frac{k \cdot P_{u,v}(\theta_m)}{d_{max} N_0} \geq \beta\)
8. \(\theta_m(u) = \theta_{max}\)
9. \(S_u = \{ \theta \in S | \theta \leq \theta_{max} \}\)
10. Sort elements \(S_u\) in descending order
11. for each beam-width \(\phi \in S_u\)
12. \(K = \frac{360}{\phi}\)
13. \(\alpha_k = k \times \phi, k = 0, \ldots, K-1\)
14. for each node \(v \in V_u\)
15. \(\Phi_{uv} = \text{relative angle of } v \text{ w.r.t. } u\)
16. \(\psi_{uv} = \text{arg min}_{\alpha_k} |\Phi_{uv} - \alpha_k|, k = 0..K-1\)
17. if \(\psi_{uv}, v \in V_u\) are all different
18. \(\theta_m(u) = \phi;\)
19. break
20. else
21. continue
22. end
23. end
24. end
25. Output: \(\theta_m(u) \forall u \in V\)