LIGHT-TREES FOR OPTICAL NETWORKS: OPTIMIZATION

PROBLEM FORMULATIONS FOR UNICAST AND

BROADCAST TRAFFIC

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Abstract

In a wavelength-routed optical network [1], a lightpath is a point-to-point all-optical wavelength channel connecting a transmitter at a source node to a receiver at a destination node. A light-tree is a point-to-multipoint generalization of a lightpath. An optimum light-tree based virtual topology has lower average packet hop distance and requires fewer opto-electronic components when compared with an optimum light-path based virtual topology [2], [3]. This study presents mathematical formulations of the optimum light-tree based virtual topology design problems for unicast and broadcast traffic.

I. INTRODUCTION

A light-tree is a point-to-multipoint all-optical channel which may span multiple fiber links. Hence, a light-tree enables “single-hop” communication between a “source” node and a set of “destination” nodes. Thus, a light-tree-based virtual topology can significantly reduce the hop distance [2], [3], thereby increasing the overall network throughput.

Fig. 1 shows a light-tree (thick lines) which connects node UT to nodes TX, NE, and IL. Thus, an optical signal transmitted by node UT travels down the light-tree till it reaches node CO where it is “split” by an “optical splitter” into two identical copies. One copy of the optical signal is routed to node TX, where it is terminated at a receiver. The other copy of the optical signal is routed towards node NE, where it is again split into two identical copies. At node NE, one copy of the optical signal is terminated at a receiver, while the other copy is routed towards node IL. (For a discussion of node architectures and optical devices which facilitate these operations, please see [2], [3].)

Finally, a copy of the optical signal reaches node IL, where it is terminated at a receiver. Thus, the “virtual topology” induced by this light-tree consists of three logical links as shown in Fig. 2. (The link labels in Fig. 2 indicate the fraction of the source node’s traffic that is destined for the link destination. The sum of these labels should be no larger than unity.)

We formulate the light-tree-based virtual topology design problem as an optimization problem with one of two possible objective functions: for a given traffic matrix, (i) minimize the network-wide average packet hop distance, or (ii) minimize the total number of transceivers in the network. We consider two types of traffic: (i) unicast and (ii) broadcast. Our mathematical formulation for these optimization problems turns out to be a mixed-integer linear program (MILP). Because of space limitation, our discussions here are brief; please see [2], [3] for more details.

II. FORMULATION OF THE OPTIMIZATION PROBLEM: UNICAST TRAFFIC

By extending the work in [4], we formulate the problem of finding an optimum light-path-based virtual topology as an optimization problem, using principles of multicommodity flow for routing of light-trees on the physical topology and for routing of packets on the virtual topology, and using the following notation: 1. s and d used as subscript or superscript denote source and destination of a packet, respectively; 2. i and j denote the originating node (or root) and the terminating node (or leaf) in a light-tree, respec-

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tively; and

3. \(m \) and \(n\) denote endpoints of a physical link that might occur in a light-tree.

- **Given:**
  - Number of nodes in the network = \(N\).
  - Maximum number of wavelengths per fiber = \(W\) (a system-wide parameter).
  - Physical topology \(P_{mn}\), where \(P_{mn} = P_{nm} = f\) (i.e., fiber links are assumed to be bidirectional) indicates that there are \(f\) direct physical fiber links between nodes \(m \) and \(n\), where \(f = 0, 1, 2, \ldots\) and \(m,n = 1, 2, 3, \ldots, N\). If there is no fiber link between nodes \(m \) and \(n\), then \(P_{mn} = P_{nm} = 0\).
  - Number of transmitters at node \(i\) = \(T_i\).
  - Number of receivers at node \(j\) = \(R_j\).
  - Traffic matrix \(A_{sd}\) denoting average rate of traffic flow from node \(s\) to node \(d\), for \(s, d = 1, 2, \ldots, N\).
  - Maximum loading per channel = \(\beta\), \(0 < \beta < 1\). \(\beta\) restricts the queuing delay on a lightpath from getting unbounded by avoiding excessive link congestion. We do not incorporate queuing delays explicitly in the problem formulation, under the assumption that they are negligible for suitable chosen values of \(\beta\). Previous results [5] indicate that queuing delays are negligible compared to propagation delays for a large network, such as the NSFNET, except under extremely heavy loading.
  - Capacity of each channel = \(C\) (normally expressed in bits/second, but converted to units of packets/second by knowing the mean packet length).

- **Variables:**
  - Number of “busy” transmitters: Variable \(V_i = 0, 1, 2, \ldots, T_i\) is the number of transmitters in use at node \(i\).
  - Virtual topology: The variable \(s_{ij} > 0\) if a transmitter at node \(i\) and a receiver at node \(j\) are members of the same light-tree in the virtual topology; \(s_{ij} = 0\) otherwise. Note that \(s_{ij}\) is a real-valued variable which can range from 0 to \(V_i\), where \(V_i\) is the number of “busy” transmitters at node \(i\). For example, \(s_{ij} = 0.5\) implies that the virtual link \((i, j)\) can use only 0.5 times the capacity of a light-tree channel. The integer-valued variable \(V_{ij}\) is used to take the “ceiling” (i.e., \(V_{ij} \geq s_{ij}\)) of the real-valued variable \(s_{ij}\). Thus, if \(V_{ij} > 0\), then link \((i, j)\) exists in the virtual topology. Note that, if \(s_{ij} > 0\) and \(s_{ij} = V_{ij}\), then node \(i\) is connected to node \(j\) by a lightpath, a confirmation of the fact that a light-tree is a generalization of a lightpath.

- Traffic routing: The variable \(\lambda_{ij}^{sd}\) denotes the traffic flowing from node \(s\) to node \(d\) and employing \(s_{ij}\) as an intermediate virtual link. Note that traffic from node \(s\) to node \(d\) may be “bifurcated” with different components taking different sets of light-trees.

- Physical topology route: The variable \(p_{mn}^{i'} > 0\) if the fiber link \(P_{mn}\) is present in a light-tree rooted at node \(i\); \(p_{mn}^{i'} = 0\) otherwise. Note that \(p_{mn}^{i'}\) is a real-valued variable which can range from 0 to \(V_i\), where \(V_i\) is the number of “busy” transmitters at node \(i\). For example, in Fig. 1, for the light-tree rooted at node \(UT\), \(p_{UT,CO}^{UT}\) = 1 and \(p_{CO,TX}^{UT} = 0.5\). A fractional value of \(p_{mn}^{i'}\), say \(p_{CO,TX}^{UT} = 0.5\), means the following: from the total amount of traffic transmitted on a light-tree rooted at node \(i\), the physical fiber link \(m-n\) can carry an amount of traffic which is less than 0.5 times the capacity of a light-tree channel.

- Dummy integer variable: Variable \(Q_{mn}^{i'} \geq p_{mn}^{i'}\) is employed to take the “ceiling” of variable \(p_{mn}^{i'}\).

- **Optimize:** Optimize one of two possible objective functions shown below.

1. Average packet hop distance:

   \[
   \text{Minimize} : \quad \frac{1}{\sum_{s,d} A_{sd}} \sum_{i,j} \sum_{s,d} \lambda_{ij}^{sd} \quad (1)
   \]

   This is a linear objective function because \(\sum_{i,j} \sum_{s,d} \lambda_{ij}^{sd}\) is a linear sum of variables, while \(\sum_{s,d} A_{sd}\) is a constant for a given traffic matrix.

2. Total number of transceivers in the network:

   \[
   \text{Minimize} : \quad \sum_i V_i + \sum_{i,j} V_{ij} \quad (2)
   \]

\(\sum_i V_i\) is the total number of transmitters in the network, while \(\sum_{i,j} V_{ij}\) is the total number of receivers in the network. Thus, the objective function minimizes the total number of transceivers in the network.

- **Constraints:**

  - On virtual topology connection variables:

    \[
    V_i \leq T_i \quad (3)
    \]

    \[
    s_{ij} \leq V_{ij} \quad (4)
    \]

    \[
    V_{ij} \leq s_{ij} + 1 \quad (5)
    \]

    \[
    \sum_j V_{ij} \geq V_i \quad (6)
    \]
\[
\sum_i V_{ij} \leq R_j \quad \text{(7)}
\]
\[
\text{int} \quad V_i, V_{ij} \quad \text{(8)}
\]

- On physical route variables:
  \[
  \sum_n p_{in}^j = V_i \quad \text{(9)}
  \]
  \[
  \sum_j s_{ij} = V_i \quad \text{(10)}
  \]
  \[
  \sum_m p_{imk} = \sum_n p_{in}^j + s_{ik}, \; k \neq i \quad \text{(11)}
  \]
  \[
  p_{imn} \leq Q_{mn}^i \quad \text{(12)}
  \]
  \[
  Q_{mn}^i \leq V_i \quad \text{(13)}
  \]
  \[
  \sum_j Q_{mn}^i \leq W \times P_{mn} \quad \text{(14)}
  \]
  \[
  \text{int} \quad Q_{mn}^i \quad \text{(15)}
  \]

- On virtual topology traffic variables \( \lambda_{ij}^{sd} \):
  \[
  \lambda_{ij}^{sd} \geq 0 \quad \text{(16)}
  \]
  \[
  \sum_j \lambda_{ij}^{sd} = \Lambda_{sd} \quad \text{(17)}
  \]
  \[
  \sum_i \lambda_{ij}^{sd} = \Lambda_{sd} \quad \text{(18)}
  \]
  \[
  \sum_k \lambda_{ij}^{sd} = \sum_j \lambda_{ij}^{sd} \quad \text{if} \quad k \neq s, d \quad \text{(19)}
  \]
  \[
  \sum_{s,d} \lambda_{ij}^{sd} \leq s_{ij} \times \beta \times C \quad \text{(20)}
  \]

- Optional constraints: physical topology as a subset of virtual topology:
  \[
  P_{mn} = 1 \Rightarrow V_{mn} \geq 1, P_{mn}^m \geq 1, Q_{mn}^m \geq 1 \quad \text{(21)}
  \]

III. FORMULATION OF THE OPTIMIZATION PROBLEM: BROADCAST TRAFFIC

In case of broadcast traffic, we are mainly interested in finding a set of light-trees which implements a "broadcast layer" over a single wavelength, e.g., for control and management of signalling information.

The formulation of the problem for broadcast traffic is similar to the formulation of the problem for unicast traffic except for the routing of traffic on the virtual topology. The equations for routing broadcast traffic consists of two parts: (i) equations for constructing a spanning tree on the virtual topology for each node in the network; thus, the spanning tree rooted at node \( s \) will route the broadcast traffic originating at node \( s \), and (ii) equations for the capacity constraint of the virtual links. The formulation is as follows.

**Variables:**
- Definitions of the variables \( V_i, T_i, s_{ij}, p_{imn}^i, V_{ij} \), and \( Q_{imn} \) remain unchanged.
- Spanning-tree variable \( 0 \leq \lambda_{ij}^s \leq N \), which determines the spanning tree rooted at node \( s \). \( \lambda_{ij}^s > 0 \) if virtual link \( V_{ij} \) belongs to the spanning tree rooted at node \( s \). Note that variable \( \lambda_{ij}^s \) is used only to "construct" a spanning tree on the virtual topology; it does not represent the broadcast traffic flow on the spanning tree. The traffic on each link of a spanning tree which is rooted at node \( s \) is equal to the amount of broadcast traffic that is sourced at node \( s \) (denoted by \( \Lambda_s \)). Variable \( 0 \leq \lambda_{ij}^s \leq 1 \) is a 0-1 integer variable, such that \( \lambda_{ij}^s = 1 \) if virtual link \( V_{ij} \) belongs to the spanning tree rooted at node \( s \); otherwise, \( \lambda_{ij}^s = 0 \).
- Broadcast traffic \( \Lambda^s \) originating from node \( s \).

**Given:**
- Definitions of the constants \( N, W, P_{mn}, T_i, R_i, \beta, \) and \( C \) remain unchanged.
- Broadcast traffic \( \Lambda^s \) denotes the average rate of broadcast traffic originating from source \( s \), where \( s = 1, 2, 3, \ldots, N \).

**Constraints:**
- On virtual topology connection variables: same as in the previous section.
- On physical route variables: same as in the previous section.
- On “spanning tree” variables \( \lambda_{ij}^s \) and \( \Lambda_{ij}^s \):
  \[
  \sum_j \lambda_{ij}^s = N - 1 \quad \text{(22)}
  \]
  \[
  \sum_m \lambda_{imk}^s = \sum_n \lambda_{kn}^s + 1, k \neq s \quad \text{(23)}
  \]
  \[
  \Lambda_{ij}^s \leq V_{ij} \quad \text{(24)}
  \]
  \[
  \lambda_{ij}^s \leq (N - 1) \times \Lambda_{ij}^s \quad \text{(25)}
  \]
  \[
  \sum_i \lambda_{ij}^s \leq N - 1 \quad \text{(26)}
  \]
  \[
  \sum_k \Lambda_{ij}^k \leq s_{ij} \times \beta \times C \quad \text{(27)}
  \]

**Optimize:**
\[
\text{Minimize :} \quad \sum_i V_i + \sum_{i,j} V_{ij} \quad \text{(28)}
\]
IV. RESULTS

Table I compares the network-wide average hop distance for a lightpath-based virtual topology vs. a light-tree-based virtual topology using a MILP solver, CPLEX. The traffic matrix is randomly generated on the NSFNET backbone which is used as our physical topology. As expected, the average packet hop distance in a light-tree-based virtual topology is much lower than the average packet hop distance in a lightpath-based virtual topology, thereby demonstrating the advantage of using light-trees.

Table II compares the number of transceivers (optoelectronic components) required in a light-tree-based virtual topology vs. the number of transceivers required in a lightpath-based virtual topology. It is interesting to note that, if we use random traffic matrices, then the number of transceivers required by a light-tree-based solution is only a little lower than the number of transceivers required by a lightpath-based solution. On the other hand, if we use an “ordered”, i.e., non-random traffic matrix, then the number of transceivers required in a light-tree-based solution may be significantly less. For example, consider the following traffic matrix: every node sends a fixed amount of data to its neighbors and does not send any data to any other node in the network. Note that the traffic matrix of a network control layer may have this pattern. Table II shows that for such a “one-hop” traffic matrix, a light-tree-based solution requires significantly fewer transceivers. Please see [2], [3] for more details.

<table>
<thead>
<tr>
<th>Transceivers</th>
<th>Wavelengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.59 1.58 1.58</td>
</tr>
<tr>
<td>5</td>
<td>1.48 1.38 1.38</td>
</tr>
<tr>
<td>6</td>
<td>1.42 1.32 1.30</td>
</tr>
</tbody>
</table>

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</thead>
<tbody>
<tr>
<td>4</td>
<td>1.23 1.13 1.08</td>
</tr>
<tr>
<td>5</td>
<td>1.21 1.12 1.07</td>
</tr>
<tr>
<td>6</td>
<td>1.19 1.09 1.07</td>
</tr>
</tbody>
</table>

TABLE I
AVERAGE PACKET HOP DISTANCE FOR A LIGHTPATH-BASED VIRTUAL TOPOLOGY AND A LIGHT-TREE-BASED VIRTUAL TOPOLOGY.

<table>
<thead>
<tr>
<th>Traffic Matrix</th>
<th>Virtual Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>104</td>
</tr>
<tr>
<td>“One-Hop”</td>
<td>54</td>
</tr>
</tbody>
</table>

TABLE II
THE NUMBER OF TRANSCIEVERS REQUIRED BY A LIGHTPATH-BASED VIRTUAL TOPOLOGY AND A LIGHT-TREE-BASED VIRTUAL TOPOLOGY FOR DIFFERENT TRAFFIC MATRICES.

Fig. 1. NSFNET backbone topology. (Link labels correspond to propagation delays.)

Fig. 2. Virtual links induced by the light-tree consisting of nodes UT, NE, TX, and IL.

REFERENCES