Multiple-dimensional Markov Chain Model on Channel Bonding

Speaker: Lin Wang

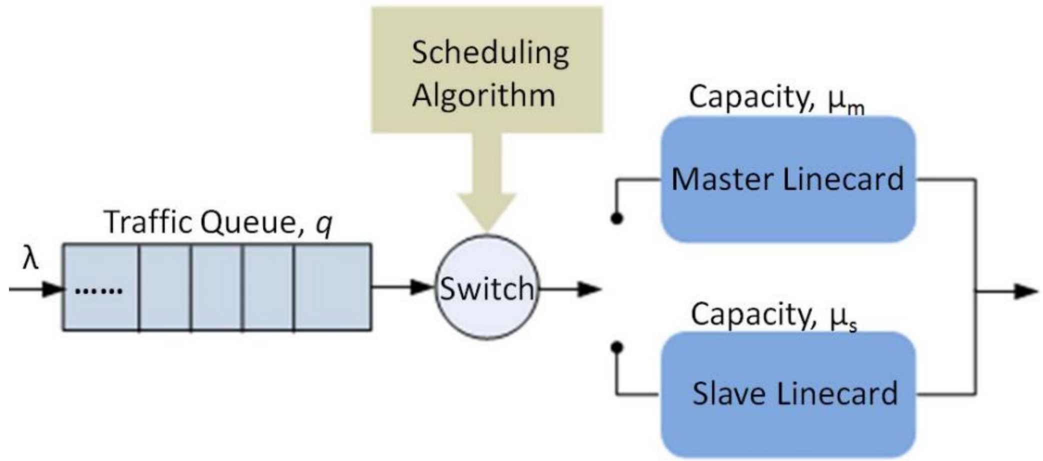
Research Advisor: Biswanath Mukherjee

UCDAVIS

Paper 1: Improving Energy-Efficiency of HFC Networks with a Master-Slave Linecard Configuration

System Model

A CMTS chassis equipped with two linecards (LC) and share one buffer.



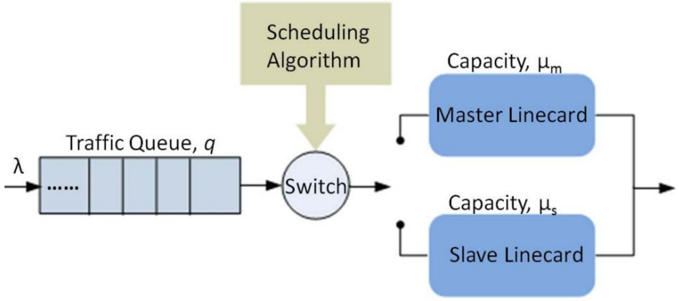
Ping, Lu, Yuan Yabo, and Zhu Zuqing. "Improving Energy-Efficiency of HFC Networks with a Master-Slave Linecard Configuration.“
Zhu, Zuqing. "Design of energy-saving algorithms for hybrid fiber coaxial networks based on the DOCSIS 3.0 standard." Journal of Optical Communications and Networking 4.6 (2012): 449-456.

Assumption

1. Packets arrive at Poisson process with rate λ;
2. Buffer is finite with number N;
3. L(t) is # of packets in the queue at time t;
4. Service rate ~ exp(μ_m) and exp(μ_s) for master and slave LCs;
5. LC operation modes:
   Working: P_m or P_s;
   Sleeping: P_0;

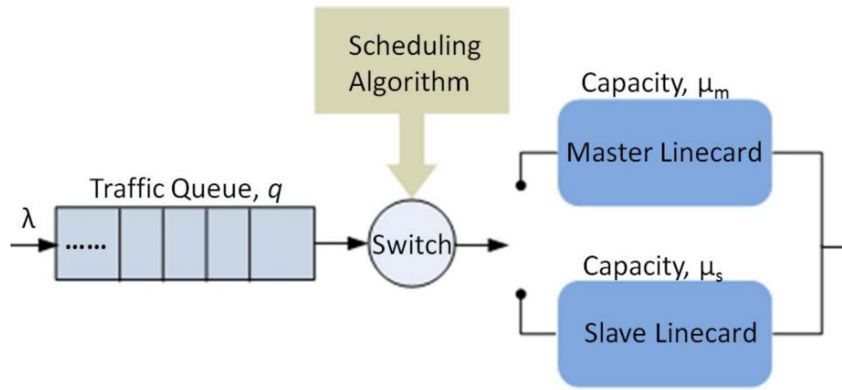
$$P = \alpha \cdot \mu^{2/3}$$

6. Traffic queue sampling period ~ exp(S);
7. LC switching threshold: δ;
8. If L(t)=0 and master LC is idle, then LC switch happens.



Algorithm 1 Energy-Efficient Scheduling for Master-Slave Configuration

while system is operational **do**
    measure $L(t)$;
    if $L(t) > \mu_s + \varepsilon$ **then**
        if slave LC is in working mode **then**
            invoke a LC switch: slave $\mapsto$ master;
            put slave LC into sleeping mode;
        end if
    else if $L(t) \leq \mu_s - \varepsilon$ **then**
        if master LC is in working mode **then**
            invoke a LC switch: master $\mapsto$ slave;
            put master LC into sleeping mode;
        end if
    end if
    determine the next queue sampling period T_s;
    $wait(T_s)$;
end while



Theoretical Analysis

- **Model the system with two-dimensional Markov process**

$\{(L(t), K(t)), t > 0\}$

L(t): # of packets in the queue;
K(t)=1: master work, slave sleep;
=2: master sleep, slave work;

$\ell(L(t))$

$$\ell(i) = \begin{cases} \{(0,2)\}, & i = 0 \\ \{(i,1),(i,2)\}, & i = 1, \ldots, N \end{cases}$$

Traffic load: $\rho = \lambda / \mu_m$

Theoretical Analysis

Markov process can be written as:

$$\mathbb{Q} = \begin{bmatrix} B_0 & C_0 & & & \\ A_1 & B_1 & C_1 & & \\ & \ddots & \ddots & \ddots & \\ & & A_{N-1} & B_{N-1} & C_{N-1} \\ & & & A_N & B_N \end{bmatrix}$$

A_i denotes the backward transition rates from level $\ell(i)$ to $\ell(i-1)$
B_i is for the local transition rates within level $\ell(i)$
C_i forward transition rates from level $\ell(i)$ to $\ell(i+1)$

$$A_i = \begin{cases} \begin{bmatrix} \mu_m \\ \mu_s \end{bmatrix}, & i = 1 \\ \begin{bmatrix} \mu_m & \\ & \mu_s \end{bmatrix}, & i \in (1, N] \end{cases}$$

$$C_i = \begin{cases} \begin{bmatrix} 0 & \lambda \end{bmatrix}, & i = 0 \\ \begin{bmatrix} \lambda & \\ & \lambda \end{bmatrix}, & i \in (0, N-1] \end{cases}$$

$$B_i = \begin{cases} -\lambda, & i = 0 \\ \begin{bmatrix} -\lambda - \mu_m - \theta & \theta \\ & -\lambda - \mu_s \end{bmatrix}, & i \in (0, \mu_s - \varepsilon] \\ \begin{bmatrix} -\lambda - \mu_m & \\ & -\lambda - \mu_s \end{bmatrix}, & i \in (\mu_s - \varepsilon, \mu_s + \varepsilon] \\ \begin{bmatrix} -\lambda - \mu_m & \\ \theta & -\lambda - \mu_s - \theta \end{bmatrix}, & i \in (\mu_s + \varepsilon, N - 1] \\ \begin{bmatrix} -\mu_m & \\ \theta & -\mu_s - \theta \end{bmatrix}, & i = N \end{cases}$$

Theoretical Analysis

- **Matrix Geometric Solution**

$$\boldsymbol{\pi} = (\pi_0, \boldsymbol{\pi_1}, \ldots, \boldsymbol{\pi_i}, \ldots, \boldsymbol{\pi_N}) \quad (6)$$

with $\boldsymbol{\pi_i}$ as the row-vector of the steady probabilities of level $\ell(i)$,

$$\boldsymbol{\pi_i} = \begin{cases} \pi_{0,2}, & i = 0 \\ \begin{bmatrix} \pi_{i,1} & \pi_{i,2} \end{bmatrix}, & i \in (0, N] \end{cases}$$

$$\boldsymbol{\pi}\mathbb{Q} = \mathbf{0}$$

$$\sum_{i=0}^{N} \boldsymbol{\pi_i}\mathbf{1} = 1$$

$\boldsymbol{\pi_i}$ can be analytically expressed with ρ, μ_m, μ_s, ε, and S.

UCDAVIS
UNIVERSITY OF CALIFORNIA

Performance Metrics

1) Average Packet Delay: According to the Little's Law, the average packet delay can be obtained as

$$\overline{D} = \frac{\sum_{i=1}^{N}\sum_{k=1}^{2} i \cdot \pi_{i,k}}{\rho \cdot \mu_m (1 - \sum_{k=1}^{2} \pi_{N,k})}$$

2) Linecard Switching Frequency: The LC switching frequency f can be derived as,

$$f = \frac{\sum_{i=\lceil \mu_s+\varepsilon+1 \rceil}^{N} \pi_{i,2} + \sum_{i=1}^{\lfloor \mu_s-\varepsilon \rfloor} \pi_{i,1} + S \cdot \mu_m \cdot \pi_{1,1}}{S}$$

3) Energy Efficiency Improvement: The energy-efficient improvement η, can be calculated as,

$$\begin{aligned}\eta &= 1 - \frac{P_{energy-saving}}{P_{normal}} \\ &= 1 - \frac{P_m \sum_{i=1}^{N} \pi_{i,1} + P_s \sum_{i=0}^{N} \pi_{i,2} + P_0}{P_m}\end{aligned}$$

UCDAVIS
UNIVERSITY OF CALIFORNIA

Simulation Results

N, System queue length	10000 packets
μ_m, Master LC's service rate per time unit	100 packets
μ_s, Slave LC's service rate per time unit	1 - 100 packets
P_m, Master LC's average power in working mode	1 power-unit
P_0, A LC's power in sleeping mode	0 power-unit
ρ, Traffic load	0 - 1

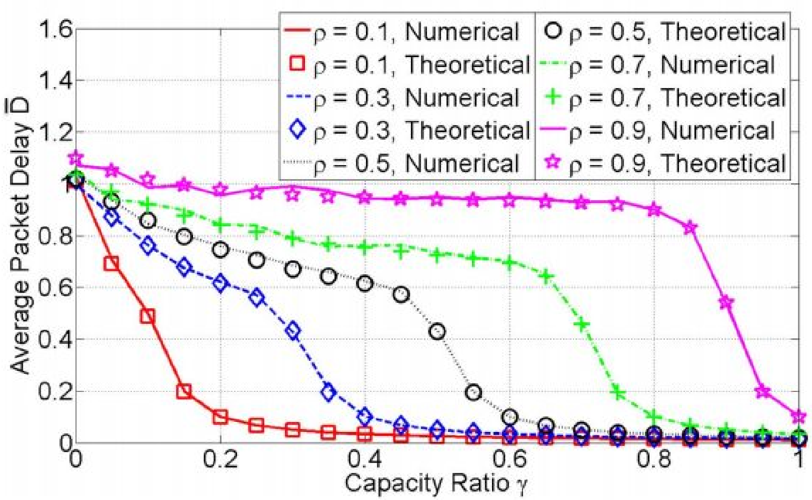


Fig. 2. Average packet delay $\overline{D}$ vs. capacity ratio γ, with $S = 1$ and $\varepsilon = 0$.

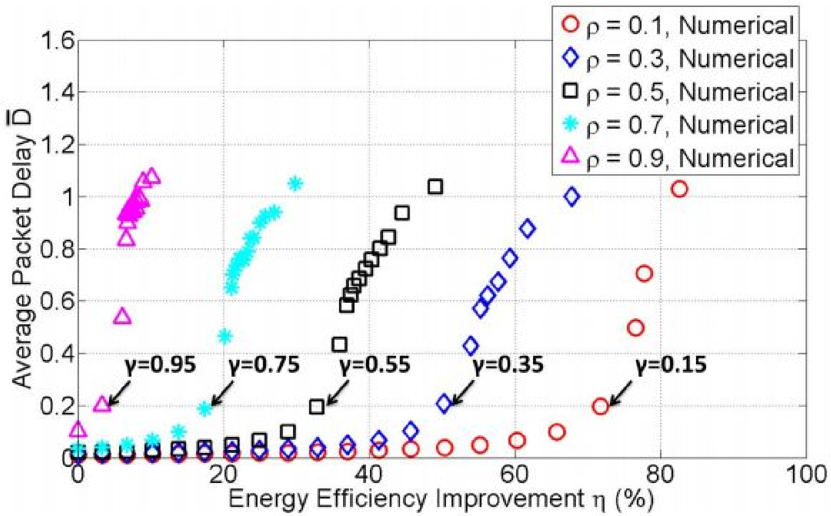
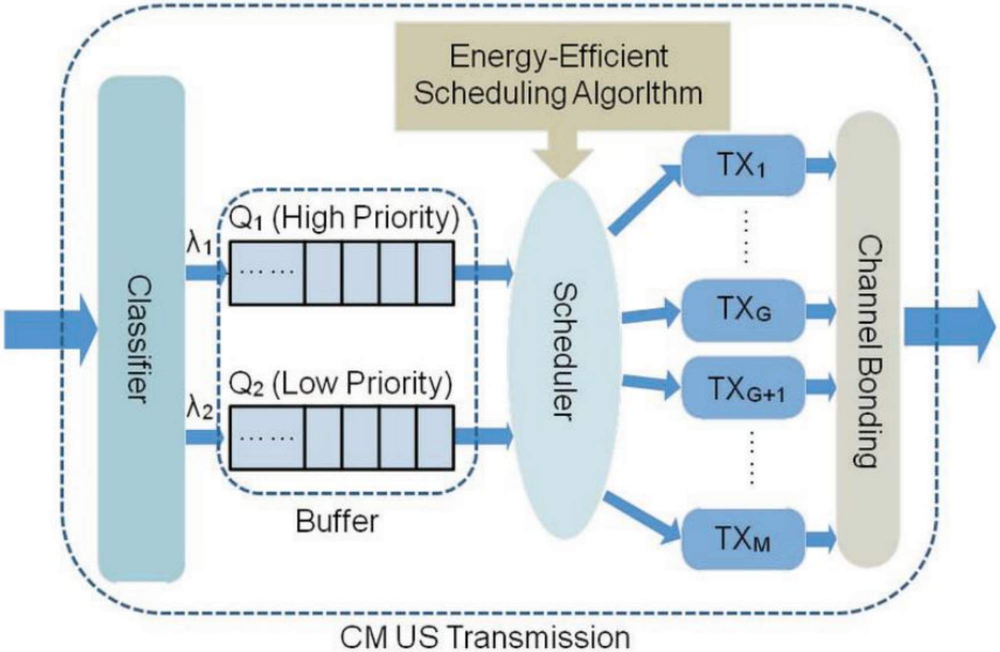


Fig. 5. Energy-delay tradeoff with $S = 1$ and $\varepsilon = 0$.

Paper 2: On the Performance Analysis of Energy-Efficient Upstream Scheduling for Hybrid Fiber-Coaxial Networks with Channel Bonding

System Model

A channel-bonding cable modem (CM) equips with multiple high speed transceivers.

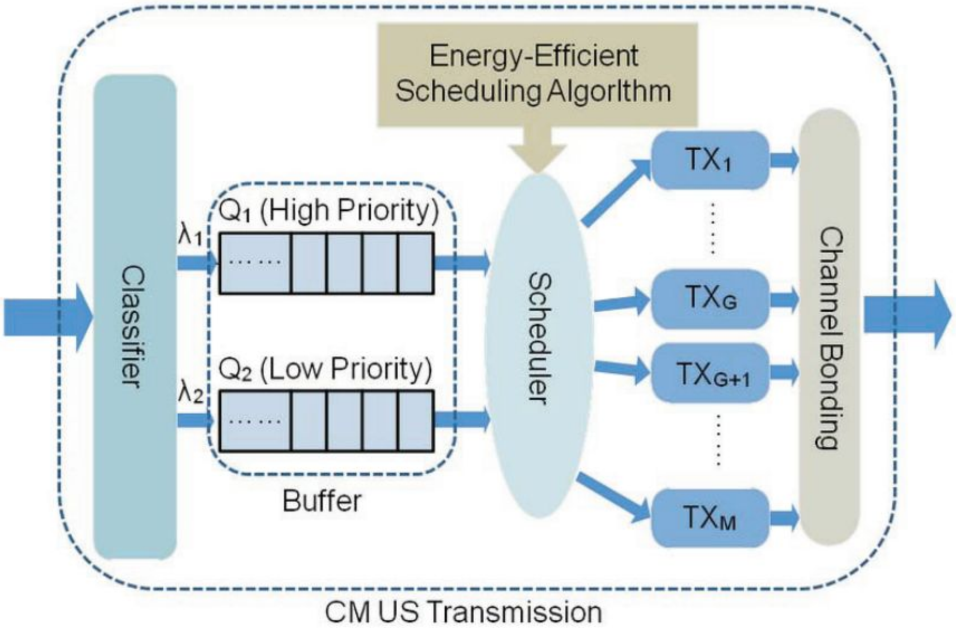


Lu, Ping, et al. "On the performance analysis of energy-efficient upstream scheduling for hybrid fiber-coaxial networks with channel bonding."*Communications Letters, IEEE* 17.5 (2013): 1020-1023.

Assumption

1. Two priority queues Q_1 and Q_2, N
2. Two traffic priorities: best effort;
   delay-sensitive;
3. $L_q(t)$ is total # of packets in the queue at time t;
4. Use *M* TXs to model multiple channel bonding, G TXs active always while the rest could sleep;
5. LC Operation modes:
   Working: P_W;
   Sleeping: P_0;
   Setting up: E_s;

7. TX's turn on threshold: T;



Theoretical Analysis

Model the system with three-dimensional Markov process

$(L_q(t), J(t), K(t))$ $J(t)$ is the number of high priority packets in Q_1

$K(t)$ is the number of working TXs at t

$$\mathfrak{Q} = \begin{bmatrix} B_0 & C_0 & & & \\ A_1 & B_1 & C_1 & & \\ & \ddots & \ddots & \ddots & \\ & & A_{N-1} & B_{N-1} & C_{N-1} \\ & & & A_N & B_N \end{bmatrix}$$

$$\boldsymbol{\pi_i} = \pi_0 * \prod_{k=1}^{i} R_i, \quad i \in [1, N],$$

where

$$R_i = \begin{cases} -C_{i-1}(B_i + R_{i+1}A_{i+1})^{-1}, & i \in [1, N) \\ -C_{N-1}B_N^{-1}, & i = N \end{cases}$$

$$\boldsymbol{\pi}\mathfrak{Q} = \mathbf{0}.$$

Performance Metrics

1) Average Packet Delays: Let L denote the average number of total packets in Q_1 and Q_2 over the operation time. With Eqs. (1) and (2), we derive L as

$$L = \sum_{i=1}^{G}\sum_{j=0}^{i} i * \pi_{(i,j,G)} + \sum_{i=G+1}^{M}\sum_{j=0}^{i}\sum_{k=G}^{i} i * \pi_{(i,j,k)} + \sum_{i=M+1}^{N}\sum_{j=0}^{i}\sum_{k=G}^{M} i * \pi_{(i,j,k)}.$$

Let L_1 denote the average number of packets in Q_1,

$$L_1 = \sum_{i=1}^{G}\sum_{j=1}^{i} j * \pi_{(i,j,G)} + \sum_{i=G+1}^{M}\sum_{j=1}^{i}\sum_{k=G}^{i} j * \pi_{(i,j,k)} + \sum_{i=M+1}^{N}\sum_{j=1}^{i}\sum_{k=G}^{M} j * \pi_{(i,j,k)}.$$

And L_2, the average number of packets in Q_2, is

$$L_2 = L - L_1$$

And L_2, the average number of packets in Q_2, is

$$L_2 = L - L_1$$

The effective arrival rates of Q_1 and Q_2 are

$$\lambda_1^e = \lambda_1 * [1 - \sum_{j=0}^{N}\sum_{k=G}^{M} \pi_{(N,j,k)}]$$

$$\lambda_2^e = \lambda_2 * [1 - \sum_{j=0}^{N}\sum_{k=G}^{M} \pi_{(N,j,k)}]$$

According to the Little's Law, the average delays packets in Q_1 and Q_2 can be obtained as

$$D_1 = \frac{L_1}{\lambda_1^e}, \quad D_2 = \frac{L_2}{\lambda_2^e}$$

Performance Metrics

2) Average Power Consumption: To calculate the frequency of setting-up modes N_s, i.e., the average number of setting-mode modes that the TXs experience in a time-unit, we define $\theta = 1/S$. Then, N_s can be obtained as

$$N_s = \begin{cases} \theta * \sum_{i=T}^{N} \sum_{j=0}^{i} \sum_{k=G}^{M} \pi_{(i,j,k)}(M-k), & T > M \\ \theta * \left\{ \sum_{i=T}^{M} \sum_{j=0}^{i} \sum_{k=G}^{i} \pi_{(i,j,k)}(M-k) + \sum_{i=M+1}^{N} \sum_{j=0}^{i} \sum_{k=G}^{M} \pi_{(i,j,k)}(M-k) \right\}, & T \leq M \end{cases} \tag{12}$$

The average power consumption of the CM is the power consumption of TXs' operation modes averaged over the corresponding steady state probability. Since the average number of working TXs in the system, N_w, can be calculated as

$$N_w = G * \sum_{i=0}^{G} \sum_{j=0}^{i} \pi_{(i,j,G)} + \sum_{i=G+1}^{M} \sum_{j=0}^{i} \sum_{k=G}^{i} k * \pi_{(i,j,k)} + \sum_{i=M+1}^{N} \sum_{j=0}^{i} \sum_{k=G}^{M} k * \pi_{(i,j,k)}, \tag{13}$$

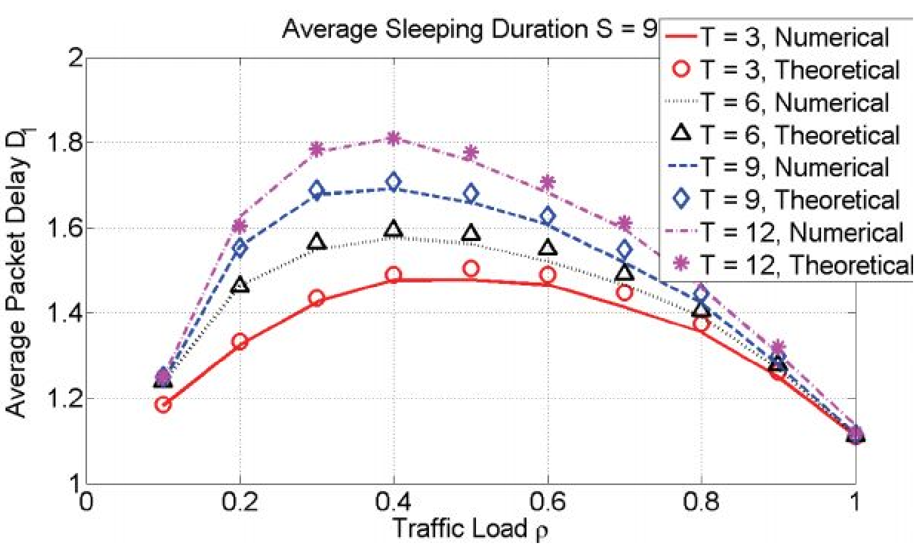
we get the average power consumption as

$$\overline{P} = P_w * N_w + P_s * (M - N_w) + E_s * N_s. \tag{14}$$

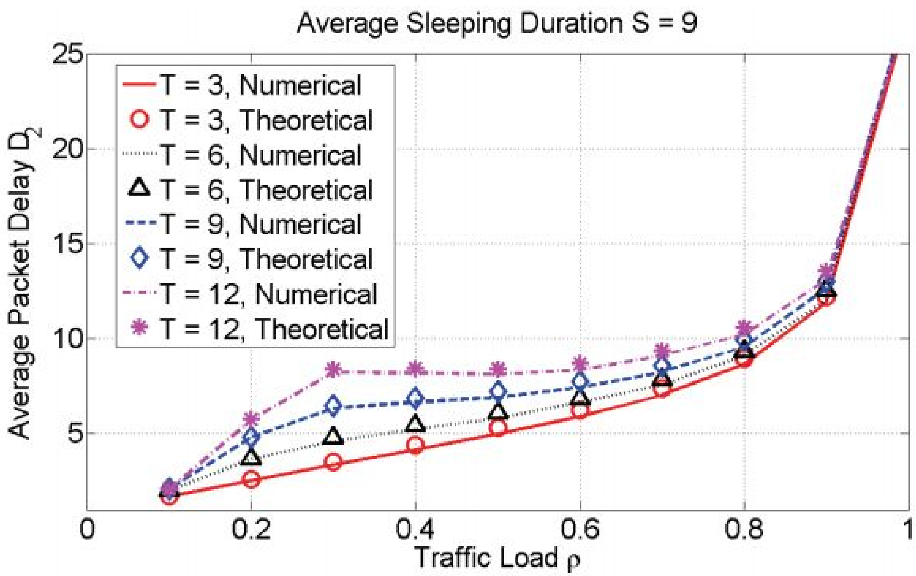
UCDAVIS
UNIVERSITY OF CALIFORNIA

Simulation Results

N, System buffer size in numbers of packets	100
M, Number of TXs in the system	4
G, Number of TXs that are normally on	1
μ, Service rate of a TX	1 time-unit/packet
ρ, Traffic load	0 - 1
P_w, TX's power consumption in working mode	1 power-unit
P_s, TX's power consumption in sleeping mode	0 power-unit
E_s, TX's energy consumption in each setting-up mode	5 energy-unit
Number of time-units in a simulation	6000
Number of simulations for statistical accuracy	50



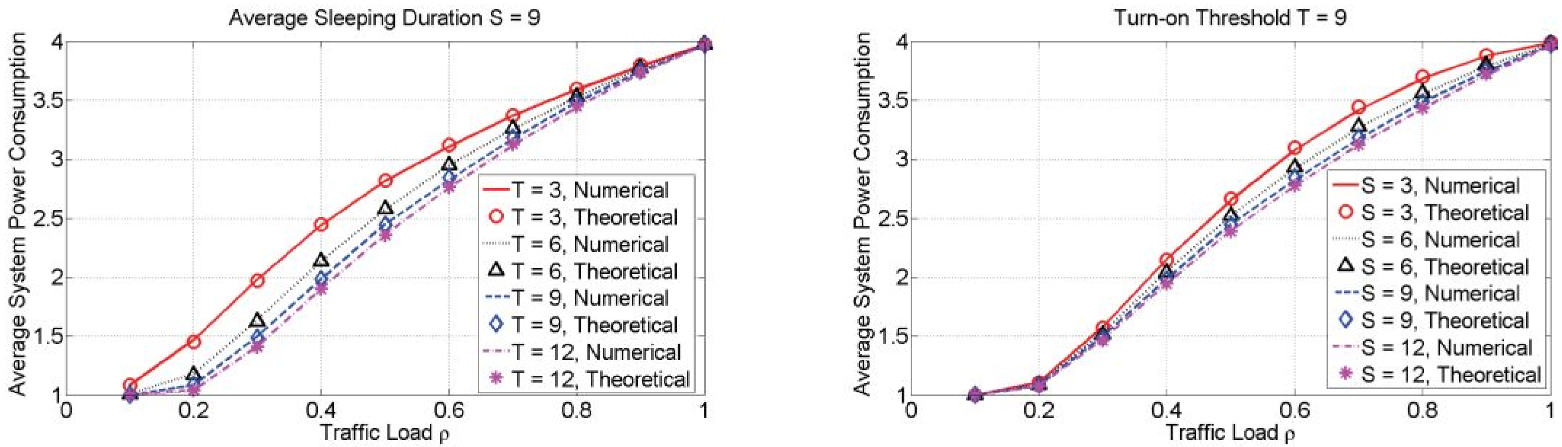
(a) Average delay of high-priority packets, D_1



(b) Average delay of low-priority packets, D_2

UCDAVIS UNIVERSITY OF CALIFORNIA

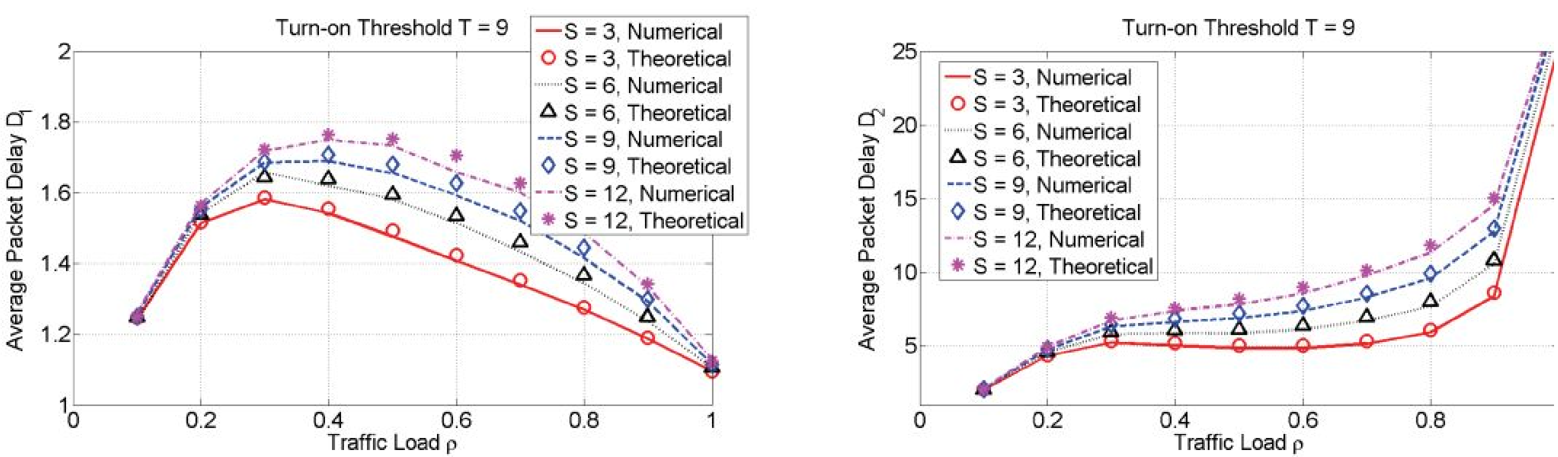
Simulation Results



(a) Impact of T with $S = 9$.

(b) Impact of S with $T = 9$.

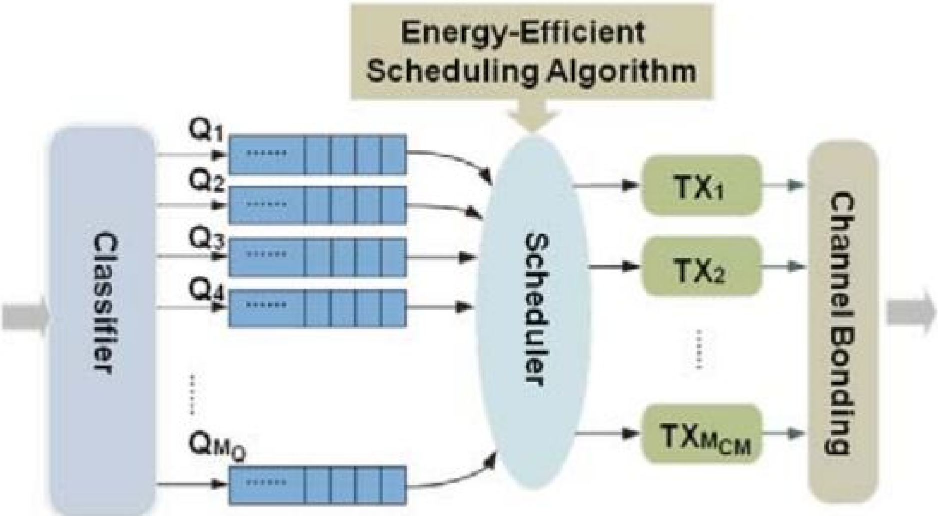
'ig. 3. Average power consumption $\overline{P}$ versus traffic load ρ.



(a) Average delay of high-priority packets, D_1

(b) Average delay of low-priority packets, D_2

Paper 3: Energy-Efficient Scheduling and Energy Delay Tradeoff in Green Hybrid Fiber-Coaxial Networks



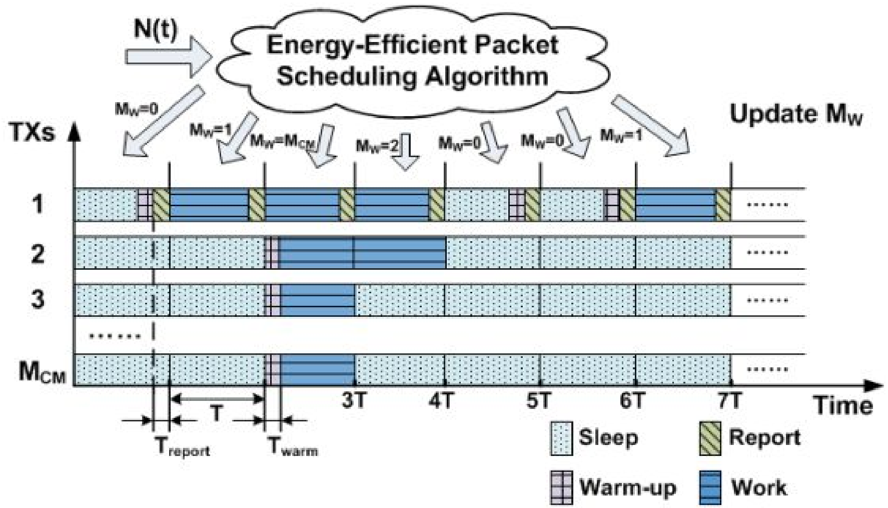


Fig. 4. Time diagram of energy-efficient traffic scheduling.

Ping, L. "Energy-efficient scheduling and energy-delay tradeoff in green hybrid fiber-coaxial networks." *Proc. of GLOBECOM 2013* (2013): 1-6.

Energy Efficient traffic scheduling algorithm

- Use Turn-on threshold N_T and # of waiting packets N(t) to decide the number of working TXs for next scheduling cycle.

- Send control message to TX channels, and each channel sets its operation mode as working or sleeping.

- If a TX has been in sleeping mode for Kmax cycles, and there is waiting packets, it will be waken up anyway.

Analytical Derivations

energy-consumption in a Scheduling Cycle is

$$\overline{E_{norm}} = M_{CM} \cdot P_{work} \cdot T$$

$$\overline{E_{eff}} = \sum_{m=0}^{M_{CM}} Prob(m)\overline{E(m)}$$

$$\begin{aligned}\overline{E_{eff}} &\approx M_{CM}T((1-\rho)P_{sleep} + \rho P_{work}) \\ &+T_{report}(P_{work} - P_{sleep})\end{aligned}$$

In M/D/m queue, average delay can be estimated by M/M/m queue

$$\begin{aligned}\overline{D^{M/M/m}(m)} &= \frac{(\rho M_{CM})^m \mu_s}{m!(m-\rho M_{CM})} \cdot \\ &\{\frac{m-\rho M_{CM}}{m}\sum_{n=0}^{m-1}\frac{(\rho M_{CM})^n}{n!} + \frac{(\rho M_{CM})^m}{m!}\}^{-1} \quad (14)\end{aligned}$$

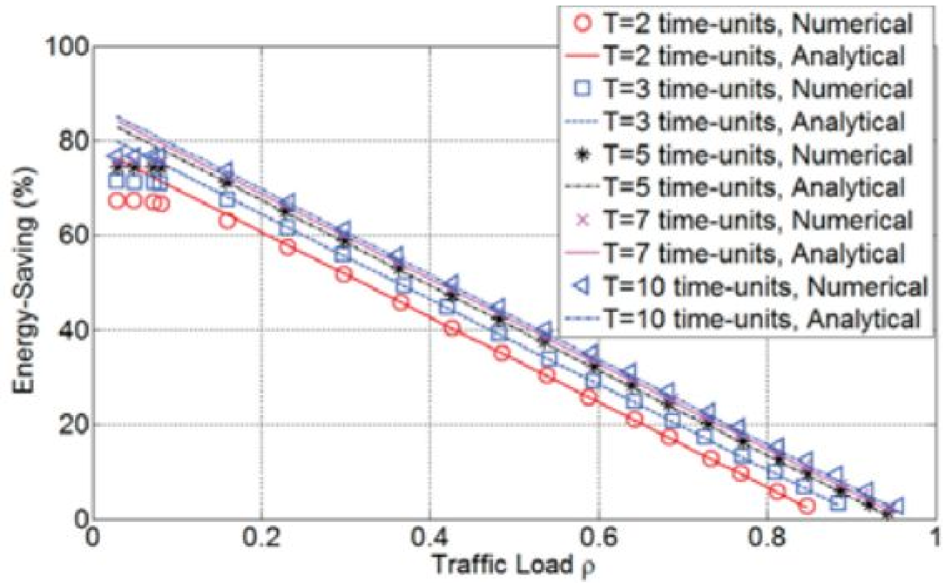
Then, we can get $\overline{D(m)}, m > 0$ as [16]

$$\begin{aligned}\overline{D(m)} &\approx \frac{\overline{D^{M/M/m}(m)}}{2} \cdot \\ &\{1 + \frac{(m-\rho M_{CM})(m-1)(\sqrt{4+5m}-2)}{16\rho m M_{CM}}\} \quad (15)\end{aligned}$$

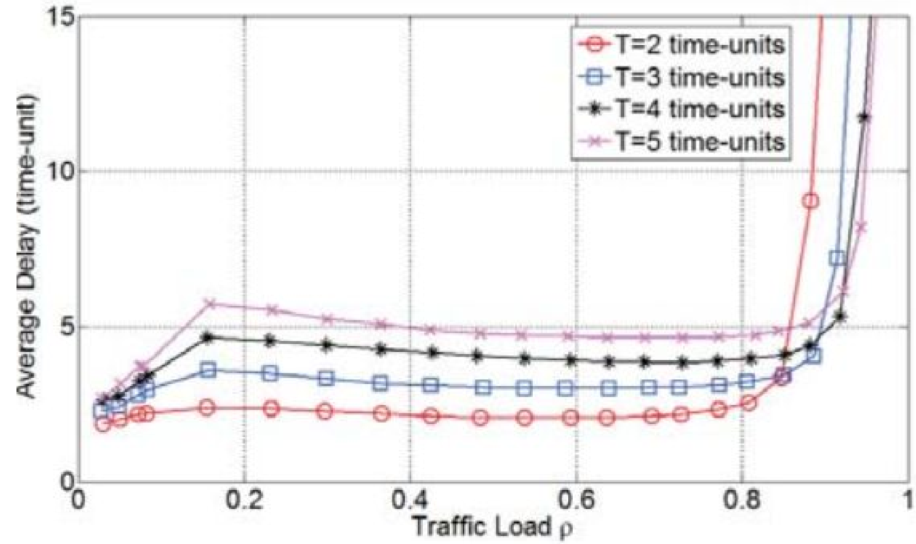
F
f

UCDAVIS
UNIVERSITY OF CALIFORNIA

Simulation Results



5. Energy-saving achieved by the energy-efficient schedu
ifferent T (single traffic priority).



5. Average delay with the energy-efficient scheduling
ent T (single traffic priority).

amlwang@ucdavis.edu