## Scalable algorithms for Jitter minimized scheduling in reconfigurable CoE

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## Reconfiguration in fronthaul

- 5G systems aim to achieve flexibility and reconfigurability in both radio access part and signal processing part
- Classified into bandwidth reconfigurability and network reconfigurability
- Bandwidth reconfigurability : flexible on-the-fly bandwidth allocation to fronthaul links depending on need of RE
- Fronthaul can be dimensioned for current traffic rather than peak traffic, saving capacity and network equipment based on traffic profile, antenna capacity, cell size, user level QoE
- Network reconfigurability : ability to change fronthaul network topology on-the-fly based on requirements of cells
- Network can change based on co-ordination scenarios (CoMP), energyefficiency schemes, etc., thus changing fronthaul topology


## Bandwidth Reconfiguration



## Bandwidth Reconfiguration - Antenna configurations

- CPRI allows rate negotiation to switch to different line rate when antenna configuration changes
- Antenna reconfiguration on/off (sectors, antenna, cell) can happen depending on user traffic requirement
- Power saving, interference reduction, frequency reuse etc. are use cases
- LTE cell breathing - adaptive coding (each sector has different coding)
- Vendor - cell adaptive (change frequency) 3GPP - each sector different modulation - within a sector different modulation ( 5 Mhz to 20Mhz)

| Antenna configuration | LTE Bandwidth |  |
| :--- | :--- | :--- |
|  | $\mathbf{c} 10 \mathrm{MHz}$ |  |
| $2 \times 2 \mathrm{MIMO}$ | $1.2288 \mathrm{Gbps}(\mathrm{IP}$ rate 75 Mbps$)$ | 2.4576 Gbps (IP rate 150 Mbps$)$ |
| $4 \times 2(4 \times 4) \mathrm{MIMO}$ | $2.4576 \mathrm{Gbps}(\mathrm{IP}$ rate 150 Mbps$)$ | $4.9152 \mathrm{Gbps}(\mathrm{IP}$ rate 300 Mbps$)$ |
| $8 \times 2(8 \times 4,8 \times 8)$ MIMO | $4.9152 \mathrm{Gbps}(\mathrm{IP}$ rate 300 Mbps$)$ | 9.8304 Gbps (IP rate 600 Mbps$)$ |
| *Source: CPRI Specification v6.0 (Aug. 30, 2013) |  |  |



| Antennas <br> for MIMO | Sectors | LTE BW <br> $(M H z)$ | IQ DR | CPRI DR | Line Rate |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $1 / 2$ | 1 | $10 / 5$ |  | 614.4 | 1 |
| $1 / 2 / 4$ | 1 | $20 / 10 / 5$ |  | 1228.8 | 2 |
| $1 / 2$ | 3 | $10 / 5$ |  | 1843.2 | 3 |
| $2 / 4$ | 1 | $20 / 10$ |  | 2457.6 | 4 |
| $2+1$ | 2 | $20+10$ |  | 3072.0 | 5 |
| $1 / 2$ | $3+3$ | $10 / 5$ | 3686.4 | 6 |  |
| $1 / 2+1$ | $3+3+1$ | $10 / 5+10$ | 4300.8 | 7 |  |
| $4 / 8$ |  | $20 / 10$ |  | 4915.2 | 8 |
| $4 / 8+1$ |  | $20 / 10+10$ | 5529.6 | 9 |  |
|  |  |  | 6144 | 10 |  |
|  |  |  | 6758.4 | 11 |  |
|  |  |  |  | 7372.8 | 12 |
|  |  |  |  | 7987.2 | 13 |
|  |  |  |  | 8601.6 | 14 |
|  |  |  |  |  | 9216 |


| $\pm 14 \mathrm{MHz}$ | $\bigcirc 1$ | $\cdots 1$ |
| :---: | :---: | :---: |
| $3 \mathrm{MHz}$ | $\sum 2$ | 芯 2 |
| ${ }^{\circ} 5 \mathrm{MHz}$ | 4 | $\sim_{3}$ |
| ${ }_{0}^{0} 10 \mathrm{MHz}$ | 8 | 4 |
| $\underline{15 \mathrm{MHz}}$ | 16 | 5 |
| 20 MHz . |  | 6 |

## Fronthaul stringent requirements and problems

- CPRI switching can be very difficult (as it is very highrate)
- Need to enable statistical multiplexing by evolving from CBR CPRI to packet-based fronthaul
- Fronthaul must be dimensioned for peak traffic rather than current traffic
- Stringent performance requirements imposed by CPRI

1) 100us of one way delay - previous study shows this is met
2) 65 ns of maximum variation in delay (i.e., jitter) - prev. study showed
3) up to 10 Gbps of throughput per RRH
4) $10^{-12}$ of maximum bit error rate.

## System Model: Could/Virtualized RAN



## Jitter

Variation in delay: two types

- Intra frame jitter - due to different cycle lengths in frame - more prominent
- Reconfiguration jitter - due to adapted line rate .Change in scheduling happens in larger timescale- will be studied now
- TDM signals are isochronous meaning that time between two consecutive bits is theoretically always same. This time is called unit interval(UI)
- Jitter is conventionally measured in unit interval peak-to-peak (Ulpp) that is, difference between maximum and minimum time intervals in units of nominal UI
- For example, for an E1 signal with a UI of 488 nanoseconds, if maximum interval were 500 nanoseconds and minimum 476, jitter would be(500476)/488 = 0.05 Ulpp


## Intra frame jitter



- Flow $1,2,3$ have ditterent rates as shown , tor example, there is jitter on flow 1 since proper scheduling policy is not enforced on the CoE switch, leading to variation in delay between consecutive flow 1 packets as shown above
- For ideal zero jitter, all the flows must be exact spaced according to their inter packet time without Ethernet encapsulation


## Exhaustive search Comb fitting algorithm

Slide combs until they perfectly fit among each otherlf they do not perfectly fit, break a tooth and put them in adjacent available slot.

```
ALGORITHM 2: COMB FITTING
Input: Schedule of CoE flows given by basic offset algorithm
Output: Non-conflicting schedule of CoE packets
Step 1: Form all possible permutations of order of flows from 1 to NF (NF! different sequences) denoted by {SE;i}
    tep 2: for each sequence SE S € {SE; }
        for j in SE i.
            initialize: matcomb as first element in {SE[ /// matcomb is temporary matrix
            matcomb = matcombine(comb }\mp@subsup{}{}{j},\mathrm{ matcomb)
            end
            Calculate
        end
                            Pick matcomb with least amount of jitter 
__LATCOMBINE SUBROUTINE
Input: comb i
Output: Combined non-conflicting schedule
Step 1: Initialize: matcomb as a matrix with length as sum lengths of comb i
Step 2: Take the longest sequence out of comb , comb and add its contents to matcomb, call the other matrix
mattemp
Step 3: Shift mattemp by the multiples of ETs to form a perfect non-conflicting schedule with matcomb
Step 4: if there is a success in this procedure
        Copy mattemp to matcomb and return matcomb
Step 5: else
            Copy the non-conflicting packets of mattemp to matcomb
            for all conflicting packets in mattemp
            Find the nearest open timeslot which can fit in the packet and update matcomb
        end
        return matcomb

\section*{Explore scalable algorithms to find Jitter minimized schedule in reconfigurable CPRI}
- JOCN work had exhaustive search which is not scalable for dynamic large fronthaul which has multiple switches scheduling CoE packets
- Distance constrained scheduling
- Pinwheel algorithm
- SMD algorithm

\section*{Distance constrained scheduling}
- A common approach to scheduling hard real-time tasks with repetitive requests is periodic task model [I], in which each task \(T_{i}\) has a period \(P_{i}\) and an execution time \(\mathrm{e}_{\mathrm{i}}\)
- \(T_{i}\) must be executed once in each of its periods
- Some real-time tasks must be executed in a (temporal) distanceconstrained manner, rather than just periodically
- Temporal distance between any two consecutive executions of a task should not be longer than a certain amount of time - within tolerable jitter

\section*{Scheduling jobs with temporal distance constraints}
- Job scheduling problems for real-time jobs with temporal distance constraints (JSD) are presented
- In JSD, start times of two related jobs must be within a given distance
- General JSD problem is NP-hard
- Define multilevel unit-time JSD (MUJSD) problem for systems with \(m\) chains of unit-time jobs in which neighboring jobs in each chain must be scheduled within c time units
- Efficient algortihms exist to solve this - o( \(\mathrm{n}^{2}\) ) time algorithm, where n is total number of jobs in system, and also an \(o\left(\mathrm{~m}^{2} \mathrm{c}^{2}\right)\)-time algorithm

\section*{Scheduling problem (single processor)}
- Given a set of jobs \(J\left\{J_{1}, J_{2} J_{n}\right\}\), in which each job \(J_{i}\) has execution time \(\mathrm{e}_{\mathrm{i}}\), ready time \(\mathrm{r}_{\mathrm{i}}\), and deadline \(\mathrm{d}_{\mathrm{i}}, 1<=\mathrm{i}<=\mathrm{n}\), job scheduling with distance constraint (JSD) problem is to find a start time function \(f\) such that for \(1<=\mathrm{i}, \mathrm{j}<=\mathrm{n}\), and \(\mathrm{i}!=\mathrm{j}\),
(1) \(f\left(J_{i}\right)>=r_{i}\),
(2) \(f\left(J_{i}\right)+e_{i}<d_{i}\), and
(3) \(\left|f\left(J_{\mathrm{i}}\right)-f\left(J_{\mathrm{j}}\right)\right|<=w\left(\mathrm{~J}_{\mathrm{i}}, \mathrm{J}_{\mathrm{j}}\right)\) distance constraint between \(\mathrm{J}_{\mathrm{i}} \mathrm{J}_{\mathrm{j}}\)

Related problems: linear array problem (LAP), bandwidth minimization problem (BMP)


The m-chain tree structure of the MUJSD problem.

\section*{SMD algorithm}
- Among all jobs remaining to be scheduled, we always pick job with largest number of successors to schedule next
- If job is a head job we schedule it at empty slot, if any, closest to and before job's deadline
- If job is a tail job with its predecessor scheduled at slot s, and if there exists any empty slot between time s and time \(s+c\), we can simply schedule job at empty slot closest to time s + c
- Otherwise we schedule tail job at slot \(s\) and then reschedule its predecessor
- Lemma: If SMD terminates successfully without reporting "unschedulable," schedule generated by algorithm is a feasible schedule for job set
- Lemma: If an MUJSD system is schedulable, then SMD will find a feasible schedule for it.

\section*{AlGorithm SMD}

Step 1. Sort the jobs into \(S_{1}, S_{2}, \ldots, S_{n}\) with nonincreasing number of successors.
Step 2. For \(i\) from 1 to \(n\) do \(\{\)
if \(S_{i}\) is a head job \(H_{j}\) then \(\operatorname{SCHED}\left(i, d_{j}, 0\right)\)
else \(\left\{\right.\) suppose \(S_{i}\) 's predecessor is scheduled at slot \(s\);
\[
\operatorname{SCHED}(i, s+c, 0) ;\}
\]
\}
```

procedure SCHED(i,t,r);
{
if t=0 then output "unschedulable" and stop
else if slot t is empty then f(Si)=t-1 /* schedule S S at slot t*/
else{
suppose slot t is now assigned to }\mp@subsup{S}{k}{}\mathrm{ ;
if (Sk is the predecessor of Si) or (r=1 and Sk is reschedulable)
then {f(Si)=t-1; /* schedule Si at slot t*/

```
                    \(\operatorname{SCHED}(k, t-1,1) ;\} / *\) reschedule \(S_{k} * /\)
            else \(\operatorname{SCHED}(i, t-1, r)\);
        \}
\}

\section*{Pinwheel scheduling algorithm}
- Definitions: \(v\) is list of \(n\) positive integers. A pinwheel schedule for \(v\) is a doubly infinite sequence drawn from labels \(\{1, \ldots, n\}\) such that each label \(i\) occurs at least once in each window of \(v_{i}\) consecutive positions.
- If such a schedule exists for \(v\), then \(v\) is schedulable. \(v\) is nondecreasing value \(\sum\left(1 / v_{i}\right)\) is density of \(v\), written \(d(v)\).
- Background: necessary but not sufficient condition for schedulability is having density at most 1. If each \(v_{i}\) is a power of 2 , then density at most 1 is sufficient.
- (Chan-Chin) If \(d(v) \leq 5 / 6\), then \(v\) is schedulable.
- Comments: earliest posing of problem showed that \(d(v) \leq 1 / 2\) is sufficient to make \(v\) schedulable. If \(v\) is schedulable, then there is a periodic pinwheel schedule for \(v\) with period at most \(\Pi v_{i}\)
- For general problem, Chan and Chin gave various algorithms that proved sufficiency of \(d(v) \leq 2 / 3\) and \(d(v) \leq .65\) This was improved to 0.7 in Chan and Chin. Fishburn and Lagarias further improved it to 0.75 .
- Decision problem of schedulability is in PSPACE. For density 1, problem is in NP but may not be NP-hard. Fast algorithms for generating schedules have also been studied.

\section*{Distance-constrained scheduling algorithms}
- \(J_{i}, J_{i 2}, J_{i 3}\), , Task \(T_{i j}\), has an execution time \(e_{i}\) and a (temporal) distance constraint \(c_{i}\).
\[
\rho(\mathbf{T})=\sum_{i=1}^{n} \rho\left(T_{i}\right)=\sum_{i=1}^{n} \frac{e_{i}}{c_{i}} .
\]
- Density thresholds (schedulability conditions) for guaranteeing a feasible schedule for a pinwheel problem instance have also derived, to be 1/2, \(13 / 20,2 / 3,0.6964\), and 0.7, for \(\mathbf{S}_{\mathrm{a}}, \mathbf{S}_{\mathrm{x}}, \mathbf{S}_{\mathrm{bc}}, \mathbf{S}_{\mathrm{by}}\), and \(\mathbf{S}_{\mathrm{xy}}\)
- \(T\), is transformed into an element, \(\mathrm{a}_{\mathrm{i}}\), in pinwheel instance, where \(\mathrm{a}_{\mathrm{i}}=\) floor ( \(\mathrm{c}_{\mathrm{i}} / \mathrm{e}_{\mathrm{i}}\) )
- Every \(e_{i}\) consecutive time slots allocated to \(i^{\text {th }}\) symbol of pinwheel instance are actually allocated to one job request of task \(T_{i}\)
- Algorithms designed for pinwheel problem are used to solve DCTS problem

\section*{Distance-constrained scheduling algorithm based on \(\mathrm{s}_{\mathrm{x}}\)}
- \(\mathbf{S x}\) first tries to find an integer \(x, a_{i} / 2<x<=a_{j}\), and specializes \(\mathbf{A}\) with respect to \(\{\mathbf{X}\}\) to get specialized multiset B
- Starting from \(x=\mathrm{a}_{1}\), down to \(x=a_{1} / 2+1, \mathbf{S x}\) specializes \(\mathbf{A}\) with respect to \(\{x\) ) and chooses an \(x\) that minimizes \(p(B)\), or chooses first \(x\) which makes \(\mathbf{P}(\mathbf{B})<=1\) (or it finds that no such integer exists)
- For example, If \(\mathbf{A}=(4,6,7,13,24,28,33\}\) is specialized with respect to \(\{4\}\), specialized multiset is \(\mathbf{B}=(4,4,4,8,16,16,32\}\) with a total density of \(33 / 32>1\), and if \(\mathbf{A}\) is specialized with respect to \(\{3\}\), specialized multiset is \(\mathbf{B}=(3,6,6,12,24,24,24)\) with a total density of \(7 / 8\). Sx will choose \(x=3\) and get \(\mathbf{B}=\{3,6,6,12,24,24,24\) )
- \(S_{R}\) is operation that is used to specialize a general DC task set
- \(S_{R}\) is a generalization of \(\mathbf{S x}\), Sr specializes \(\mathbf{C}\) with respect to \(\{r\}\), where \(r\) is real number chosen from range ( \(\mathrm{c}_{1} / 2, \mathrm{c}_{1}\) ) so that specialized task set has a minimum density increase
- \(S_{R}\) uses polynomial algorithm to find best \(r\) and then specializes distance constraint multiset C with respect to \(\{r\}\).

\section*{Scheduling algorithms}
- Scheduler Sr can schedule task sets with temporal distance constraints.
- Distance-constrained task set with \(n\) tasks can be feasibly scheduled by using Scheduler Sr as long as its total density is less than or equal to \(n\left(2^{1 / n}\right.\) -1)
- Deterministic guarantee that all tasks will meet their deadlines as long as total density is held within density threshold
- If total density of a DC task set after specialization is less than or equal to 1 , DC task set can be feasibly scheduled by Scheduler Sr
- Sr supports scheduling variable flow constraints required for Jitter minimization in fronthaul
- Multiprocessor scheduling will be explored to form schedules for large number of switches in fronthaul

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\(/^{*}\) Input: \(\mathbf{T}=\left\{T_{i}=\left(e_{i,} c_{i}\right) \mid 1 \leq i \leq n\right\}\), where \(\mathbf{T}\) is a DC task set and \(c_{i} \leq c_{j}\) for all \(i<j .{ }^{*} /\)
\(/^{*}\) Output: \(r^{*}, \Phi_{\mathbf{T}}\left(\mathrm{r}^{*}\right)\), and \(\mathbf{T}^{\prime}=\left\{T_{i}^{\prime}=\left(e_{i}, b_{i} \mid 1 \leq i \leq n\right\}\right.\), where
\(b_{i} \mid b_{j}\) for all \(i<j . * /\)
1. for \(i:=1\) to \(n\) do \(l_{i}=c_{i} / 2^{\left[\log \left(c_{i} / c_{1}\right)\right]}\);
2. sort \(\left(l_{1}, l_{2}, \ldots, l_{n}\right)\) into nondecreasing order and remove duplicates;
let \(\left(k_{1}, k_{2}, \ldots, k_{u}\right)\) be the resulting sequence;
3. for \(i:=1\) to \(n\) do put \(T_{i}\) into subset \(\tau_{l_{i}}\);
4. for \(v:=1\) to \(u\) do \(\rho\left(\tau_{k_{v}}\right):=\sum_{T_{i} \in \tau_{k_{v}}} e_{i} / c_{i}\);
5. compute \(\Phi_{\mathrm{T}}\left(k_{u}\right)\) according to (4.1);
6. for \(v:=u-1\) downto 1 do \(\Phi_{\mathrm{T}}\left(k_{v}\right):=\frac{k_{v+1}}{k_{v}} \Phi_{\mathrm{T}}\left(k_{v+1}\right)-\rho\left(\tau_{k_{v}}\right)\); /* see Lemma \(1^{* /}\)
7. find \(r^{*}\) such that \(\Phi_{T}\left(r^{*}\right)=\min _{r \in\left|k_{1}, k_{2}, \ldots, k_{u}\right|} \Phi_{\mathrm{T}}(r)\);
8. for \(i:=1\) to \(n\) do \(b_{i}:=r^{*} \cdot 2^{\left\lfloor\log \left(c_{i} / r^{*}\right)\right\rfloor}\);
9. output \(r^{*}, \Phi_{\mathrm{T}}\left(r^{*}\right)\), and \(\mathrm{T}^{\prime}=\left\{T_{i}^{\prime}=\left(e_{i}, b_{i}\right) \mid 1 \leq i \leq n\right\}\).

\section*{Polynomial time algorithm for MUJSD}
- PMD algorithm generates a feasible schedule for a schedulable MUJSD system with \(m\) job chains and distance constraint \(c\) in \(O\left(m^{2} \mathrm{c}^{2}\right)\) time
- Sort job chains and re-index them so that chain with a larger tail deadline has a larger index (ties are broken arbitrarily)
- Create a pseudochain 0 which has only one job with a deadline 0 (note that head deadlines of all other job chains are larger than 0)This pseudochain serves as a marker to trigger final cleanup process which will move jobs scheduled before time 0 to empty slots after time 0
- Step 2 sets initial tail positions of \(v\)-chains and initializes counter \(p\) which points to current job chain being scheduled,

\section*{AlGorithm PMD}

Step 1. Sort and reindex the job chains in nondecreasing tail deadline order (i.e., \(d_{i k_{i}} \leq d_{i+1, k_{i+1}}\), for \(1 \leq i<m\) );

Create a pseudochain \(B_{0}\) with \(d_{0}=0\) and \(k_{0}=0\);
Step 2. Set \(D_{1}=d_{m k_{m}}\);
For \(i=2\) to \(c\) do \(\left\{\right.\) set \(\left.D_{i}=D_{i-1}-1 ;\right\}\)
Set \(p=m\);
Step 3. Let \(y\) be the index of the v-chain with \(D_{y}=\max _{1 \leq i \leq c} D_{i}\);
If \(d_{p k_{p}} \geq D_{y}\)
then \(\left\{/ *\right.\) schedule chain \(p{ }^{*}\) /
If \(p=0\) goto Step 4 ;
initialize S-list \(S_{p}\) to be \(\left[\left(k_{p} ; D_{y} ; c\right)\right]\);
reset \(D_{y}=D_{y}-\left(k_{p}+1\right) c\);
\[
\text { set } p=p-1 ;\}
\]
else \(\{/ *\) reschedule chains */
Among scheduled chains \(p+1\) to \(m\) find a chain \(B_{i}\), and locate the job \(J_{i j}, j \geq 0\), where
(S3.1) \(J_{i j}\) is the latest job in \(B_{i}\) scheduled before \(D_{y}\), and (S3.2) \(d_{i j} \geq D_{y}\).
If \(B_{i}\) and job \(J_{i j}\) exist
then reschedule jobs \(J_{i 0}, J_{i 1}, \ldots, J_{i j} ; / *\) as in \(\S 4.3 * /\) else reset \(D_{y}=D_{y}-\left\lceil\left(D_{y}-d_{p k_{p}}\right) / c\right\rceil c\); \}

\section*{Repeat Step 3.}

Step 4. If there is any head job scheduled before time 0 then output "unschedulable"; else output the \(m\) S-lists.

\section*{MUJSD problem with}
distinct distance constraints is NP-completeWith distinct distance constraints, even if we restrict graph to a bilevel tree (not a bilevel chain tree) we can show that problem is still NP-complete.```

