Optimization Problems in Network Design

Friday, April 12, 2019

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Outline

1. Background: Optimization Problems in Network Design

2. Progress in Content Connectivity Research

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Resource Allocation in Optical Networks

- *Fixed-grid*: Routing and Wavelength Assignment (RWA) to select path and wavelength for lightpaths
- *Flex-grid*: Routing and Spectrum Allocation (RSA) including parameters of tunable transponders

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[1] E. Varvarigos and K. Christodoulopoulos, "Algorithmic Aspects in Planning Fixed and Flexible Optical Networks With Emphasis on Linear Optimization and Heuristic Techniques," *J. Lightwave Technol.*, vol. 32, pp. 681-693, 2014.

Traffic Models for Resource Allocation

- Planning phase:
  - ✓Network is empty.
  - ✓Connection requests known in advance
  - ✓Offline and static
- Operational phase:
  - ✓Connection requests arrive dynamically.
  - ✓Taking into account network current utilization
  - ✓Online and dynamic

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Optical Network Optimization Problems

- RSA and RWA are NP-complete (no polynomial time solutions)
- In next slides: A review of algorithmic techniques

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Network Optimization Problems

- Problem definition:

$$\textit{Minimize } f_0(\boldsymbol{x}) \textit{ subject to } f_i(\boldsymbol{x}) \leq b_i, i = 1, \dots, m$$

where:

  - ✓ $\boldsymbol{x} = (x_1,\ x_2,\ \dots,\ x_n)^T \in R^{n\times 1}$
  - ✓ $f_0 : R^n \rightarrow R$: optimization function
  - ✓ $f_i : R^n \rightarrow R$: constraint functions.

- Linear Programming (LP), Integer Linear Programming (ILP), Connection of ILP and LP

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Linear Programming

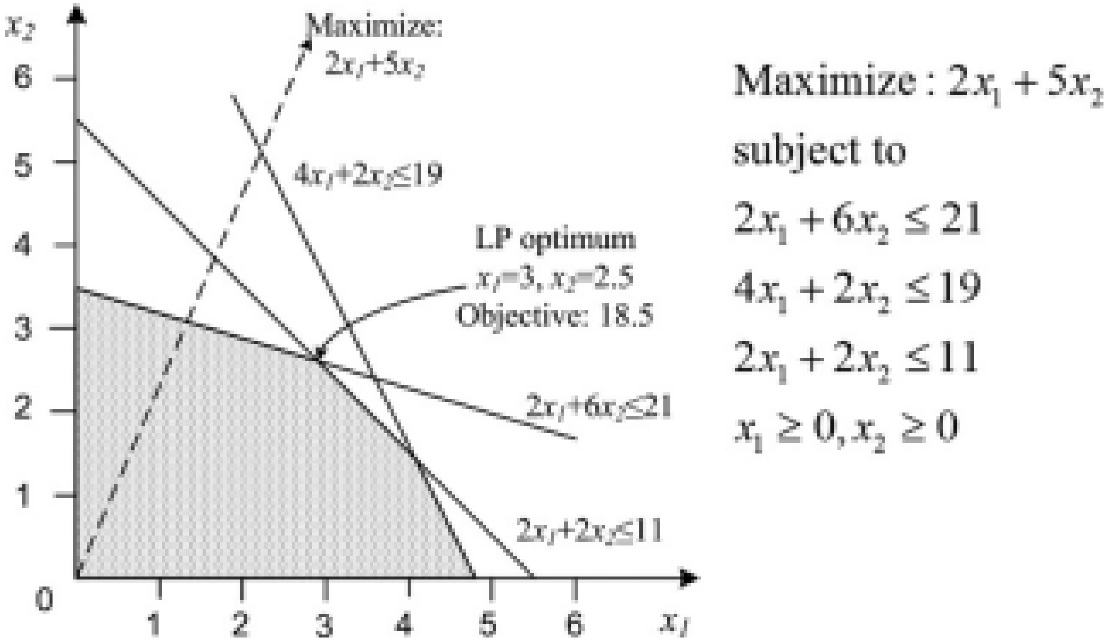
- Problem definition:

$$Minimize\ \boldsymbol{c}^T.\boldsymbol{x}\ subject\ to\ \boldsymbol{A}.\boldsymbol{x} \leq \boldsymbol{b}$$

where:

- ✓ $\boldsymbol{x} = (x_1,\ x_2,\ \dots,\ x_n)^T \in R^{n\times 1}$
- ✓ $\boldsymbol{c}: n \times 1$
- ✓ $\boldsymbol{A}: m \times n$
- ✓ $\boldsymbol{b}: m \times 1$

- Both optimization function and constraints are linear.
- An optimal solution always exists.



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Integer Linear Programming

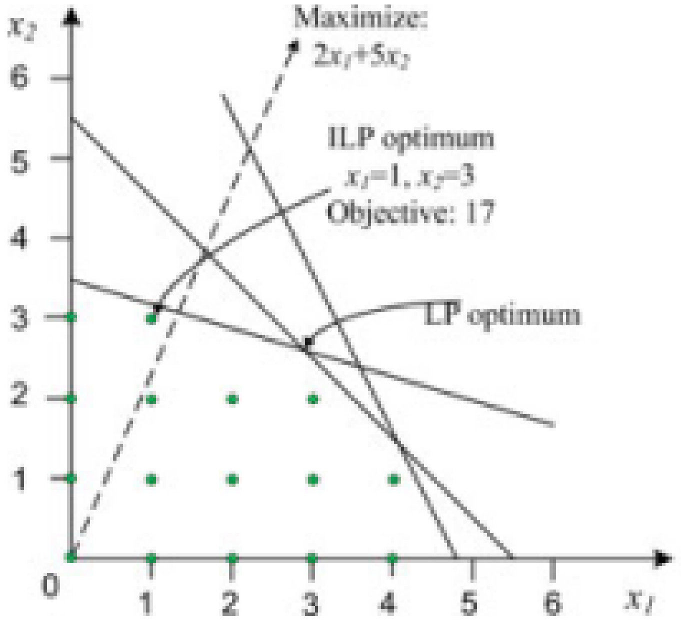
- Problem definition:

  $$Minimize\ \boldsymbol{c}^T.\boldsymbol{x}\ subject\ to\ \boldsymbol{A}.\boldsymbol{x} \leq \boldsymbol{b}$$

where:

✓ $\boldsymbol{x} = (x_1,\ x_2,\ \dots,\ x_n)^T \in Z^{n\times 1}$
✓ $\boldsymbol{c}: n \times 1$
✓ $\boldsymbol{A}: m \times n$
✓ $\boldsymbol{b}: m \times 1$

- Both optimization function and constraints are linear.
- An optimal solution is not guaranteed.



Maximize : $2x_1 + 5x_2$
subject to
$2x_1 + 6x_2 \leq 21$
$4x_1 + 2x_2 \leq 19$
$2x_1 + 2x_2 \leq 11$
$x_1 \geq 0, x_2 \geq 0$
x_1, x_2 integers

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Connection of ILP and LP

- **LP-Relaxation**:
  - ✓Solve the same ILP problem without variables to be integers.
  - ✓If solutions are integers, then they are optimal solutions.
  - ✓Providing a lower and upper bounds of optimization function
  - ✓Sophisticated techniques to increase probability the solutions are integers

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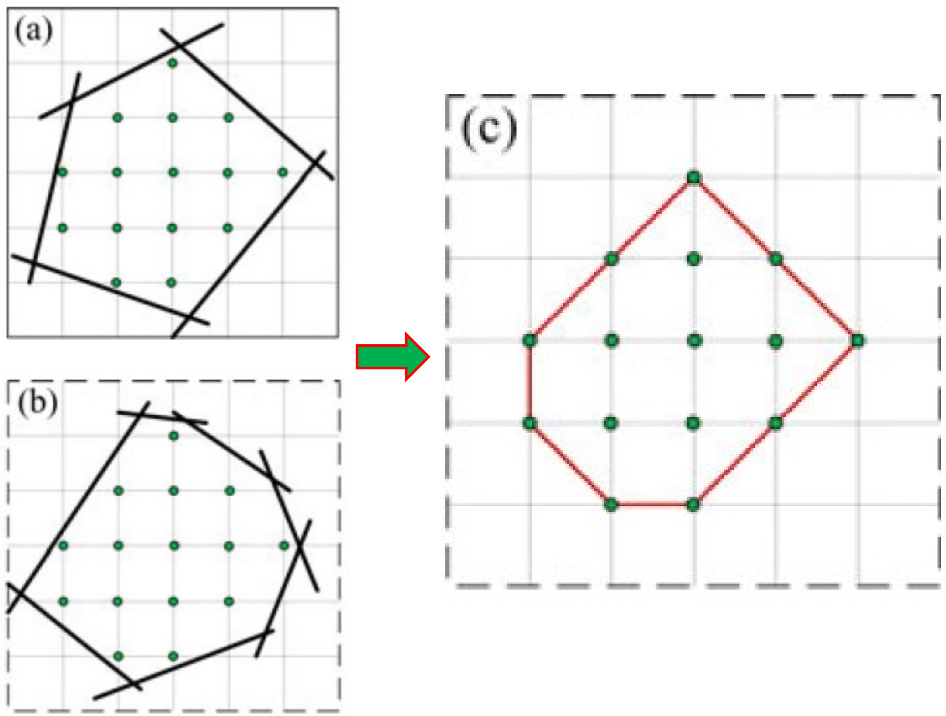
Connection of ILP and LP

- **Convex Hull**:
  - ✓Feasible solutions of an ILP can be represented by *n*-dimension polyhedrons.
  - ✓Convex hull: minimum convex set including all integer solutions
  - ✓ ILP + convex hull constraints = LP



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Connection of ILP and LP

- **Good ILP Formulations**:
  - ✓Constraints close to be a Convex Hull
  - ✓Boolean variables
  - ✓Avoid Integer and Big-M variables

Heuristic Algorithms

- For non-convex problems and NP-hard
- Local search
- Greedy algorithms
- Metaheuristic algorithms

Local Search

Algorithm 20 Local search (*best-fit*)

Initialization: Set x initial solution
do
    $next = x.$
    for each $y \in \mathcal{V}(x)$
        if $f(y) < f(x)$ **comment:** f is the cost function to minimize.
            $next = y$ **comment:** Improving solution
            comment: If *first-fit* then execute **break** here to leave the **for each** loop
while $next \neq x$ **comment:** End when no improving solution
return x

- Parameters: Vicinity Size, Connectivity of Solutions, and Best-fit or First-fit

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[2] P. P. Mariño, "Optimization of Computer Networks: Modeling and Algorithms: A Hands-On Approach," Wiley, 2016.

Simulated Annealing

Algorithm 21 Simulated annealing

Initialization:
  Set $T = T(0)$, initial temperature, x initial solution, $x_{best} = x_0$ incumbent solution
do   **comment:** Outer loop
  do   **comment:** Inner loop
    v = random chosen neighbor in $\mathcal{V}(x)$
    if $c(v) < c(x)$ **or** with probability $e^{-\frac{c(v)-c(x)}{T}}$
      $x = v$, update x_{best} if needed. **comment:** Jump to v
  while tempDecreaseCriterion
  Decrease T.
while stopCriterion
return x_{best}

- Better solution, accept; If not, still accept with the probability; High *T*, non-improving solutions are always accepted; Low *T*, non-improving solutions are never accepted.

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Tabu Search

Algorithm 22 Tabu search (basic scheme)

Initialization:
    Set x initial solution, $x_{best} = x$
    Set $\mathcal{TL} = \emptyset$ **comment:** Tabu list initially empty
do
    $x_{bestNeighbor} = \emptyset$.
    for each $y \in \mathcal{V}(x)$, y not tabu
        if $f(y) < f(x_{bestNeighbor})$
            $x_{bestNeighbor} = y$ **comment:** Improving solution
    $x = x_{bestNeighbor}$ **comment:** Jump to the best neighbor
    $\mathcal{TL} = \mathcal{TL} \bigcup a(x)$ **comment:** Update the tabu list
    if $|\mathcal{TL}| > T$ **then** remove oldest element in $\mathcal{TL}$
    if $f(x) < f(x_{best})$ **then** $x_{best} = x$ **comment:** Update incumbent solution
while stop criterion not met
return x

- Variation of Local Search permitting jumping to non-improving locations

Heuristic: Considerations

- Intensification and Diversification (clever and balanced search)
- Stop conditions: optimal bounds, max. runtime or iterations, no improvement
- Vicinity size

Network Connectivity and Content Connectivity

ILP Problem Statement

Given:

1. Physical topo.
2. Logical topo.
3. Datacenter locations
4. Wavelengths/link
5. Cut-set space (already in Logical topo.)

Cost function:

Minimize total wavelength channels

Output Results:

1. f_{NC}(vs. connectivity degree, SRG size – number of link and node failures)
2. f_{CC}(vs. connectivity degree, SRG size – number of link and node failures)
3. Result comparison
4. Running time (scalable, heuristic)

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ILP Formulation + Heuristic

```
Presolve eliminates 2693 constraints and 12890 variables.
Adjusted problem:
9176 variables, all binary
12620 constraints, all linear; 40080 nonzeros
        2080 equality constraints
        10540 inequality constraints
1 linear objective; 1810 nonzeros.


Sorry, a demo licenses for AMPL is limited to 500 variables
and 500 constraints and objectives (after presolve) for linear
problems.  You have 19608 variables, 12620 constraints, and 1 objective.
ampl: include CC1w-deg6.run;
```

In progress

1) Replicate the ILP results in AMPL + CPLEX (Online Server)

2) Develop and simulate Heuristic Algorithms for the ILP