Problem Formulation of Content Connectivity against a Double-link Failure

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Outline

- Problem formulation
- Current issues
- Solution and new directions

Note: This presentation is continuous work of the following paper:

A. Hmaity, F. Musumeci, and M. Tornatore, “Survivable virtual network mapping to provide content connectivity against double-link failures,” in *2016 12th International Conference on the Design of Reliable Communication Networks (DRCN)*, pp. 160–166, March 2016.

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Content-connected against a Double-link Failure

Problem Statement:

- ✓Minimize total number of wavelength channels
- ✓Ensure network connectivity against a single-link failure
- ✓Ensure all nodes in the logical topology can reach at least one datacenter after a double-link failure

Linear Programming (LP) Formulation

Given data: Physical Topology

- ✓ $G_P(N_P,\ E_P)$: physical topology
- ✓ N_P: set of physical nodes
- ✓ E_P: set of physical links
- ✓ If there is a link from node i to node j, so is a link from node j to node i.
- ✓ There might be more than one **fiber** on a link.

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Linear Programming (LP) Formulation

Given data: Logical Topology

- ✓ $G_L(N_L,\ E_L)$: logical topology
- ✓ N_L: set of logical nodes, $N_L \in N_P$
- ✓ E_L: set of logical **connections**
- ✓ If there is a connection from node s to node t, so from node t to node s.
- ✓ There might be more than one **lightpath** in one connection.

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Linear Programming (LP) Formulation

Given data: Other Parameters

- ✓ D: set of datacenters, $D \in N_L$
- ✓ W: number of wavelengths per fiber
- ✓ CS: complete cutset set of the logical topology (A cutset is a minimal set of logical connections whose removal disconnects the logical topology.)
- ✓ Nodes are equipped with wavelength converters.

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Linear Programming (LP) Formulation

- **Variables:**
  - ✓ $P_{ij}^{st} = V^{st} = 1$ if logical connection from s to t is mapped over physical link from i to j, otherwise 0, nonnegative integers
  - ✓ $A^{ud} = 1$ if logical node u can reach datacenter $d \in D$, otherwise 0, integers (0, 1)
  - ✓ $\alpha_{ij}^{st} = 1$ if the logical connection st fails due to a failure on physical link ij, otherwise 0, integers (0, 1)

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Linear Programming (LP) Formulation

- **Variables:**
  - ✓ $F_{ij}^{udst} = 1$ if node u can use logical connection st to reach data center d after a failure on physical link ij, otherwise 0, integers (0, 1)
  - ✓ $T_{ijkl}^{udst} = 1$ if node u can use logical connection st to reach data center d after a failure on link ij and a failure on link kl, otherwise 0, integers (0, 1)

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Linear Programming (LP) Formulation

- **Cost function:** Minimize total number of wavelength channels (a wavelength channel is a lightpath on a fiber):

$$Minimize \sum_{ij \in E_P} \sum_{st \in E_L} P_{ij}^{st}$$

Linear Programming (LP) Formulation

- **Constraints:**

1. Data center reachability: $\sum_{d \in D} A^{ud} \geq 1, \forall u \in N_L$

   - Constraint 1 guarantees that every logical node can reach at least one data center.

Linear Programming (LP) Formulation

- **Constraints:**

2. Flow conservation

$$\sum_{j:ji\in E_P} P_{ji}^{st} - \sum_{j:ij\in E_P} P_{ij}^{st} = \begin{cases} -1 \text{ if } i = s \\ 1 \text{ if } i = t \\ 0 \text{ otherwise} \end{cases}, \forall i \in N_P, \forall st \in E_L$$

➢ Constraint 2 guarantees that a flow goes out of a source node and goes into a destination node. At a transit node, input flow is equal to output flow.

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Linear Programming (LP) Formulation

- **Constraints:**

3. Path constraint

$$P_{ij}^{st} = P_{ji}^{ts}, \forall st, ts \in E_L, \forall ij, ji \in E_P$$

> ➢ Constraint 3 forces the lightpath from s to t and the lightpath from t to s to be mapped over the same path.

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Linear Programming (LP) Formulation

- **Constraints:**

4. Constraint on Alpha

    - $\frac{P_{ij}^{st}}{M} \leq \alpha_{ij}^{st} \leq P_{ij}^{st}, \forall st \in E_L, \forall ij \in E_P$

➢ Constraint 4 sets α_{ij}^{st} to 1 if the link ij is used for the connection from s to t.

Linear Programming (LP) Formulation

- **Constraints:**

5. Constraint Survivability: (Network Connectivity against a single-link failure)

$$\sum_{st \in E_L} P_{ij}^{st} + \sum_{ts \in E_L} P_{ji}^{ts} \leq |CS|, \forall\, ij, ji \in E_P, \forall CS$$

   - Constraint 5 prevents mapping all logical connections in a *cutset* over the physical links between i and j (fibers from i to j and from j to i).

Linear Programming (LP) Formulation

- **Constraints:**

6. Constraint Capacity

$$\sum_{st \in E_L} P_{ij}^{st} \leq F_{ij} W, \forall ij \in E_P$$

- Constraint 6 ensures that the total wavelength channels on each link does not exceed the maximum number lightpaths of the link.

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Linear Programming (LP) Formulation

- **Constraints:**

7. α and F relation

$$F_{ij}^{udst} + \alpha_{ij}^{st} \leq 1, \forall\, ij \in E_P, \forall st \in E_L, \forall u \in N_L, \forall d \in D$$

➢ Constraint 7 ensures that node *u* does not use logical connection *st* to reach data center *d* if the logical connection fails due to a failure on link *ij*.

Linear Programming (LP) Formulation

- **Constraints:**

8. F and T relation

$$0 \leq F_{ij}^{udst} + F_{kl}^{udst} - 2T_{ijkl}^{udst} \leq 1,$$
$$\forall ij, kl \in E_P, \forall u \in N_L, \forall d \in D, \forall st \in E_L$$

- Constraint 8 ensures that node *u* can use logical connection *st* to reach data center *d* after a failure on link *ij* and a failure on link *kl* if and only if $F_{ij}^{udst} = 1$ and $F_{kl}^{udst} = 1$.

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Linear Programming (LP) Formulation

- **Constraints:**

9. Data center reachability after a single-link failure

$$\sum_{t:st\in E_L} F_{ij}^{udst} - \sum_{t:ts\in E_L} F_{ij}^{udts} = \begin{cases} 1 - A^{ud}, u = s \wedge s = d \\ 1, u = s \wedge s \neq d \\ -A^{ud}, u \neq s \wedge s = d \\ 0, 1, u \neq s \wedge s \neq d \end{cases},$$
$$\forall u \in N_L, \forall d \in D, \forall st, ts \in E_L, \forall ij \in E_P, \forall s \in N_L$$

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Linear Programming (LP) Formulation

- **Constraints:**

10. Data center reachability after a double-link failure

$$\sum_{t:st\in E_L} T_{ijkl}^{udst} - \sum_{t:ts\in E_L} T_{ijkl}^{udts} = \begin{cases} 1 - A^{ud}, u = s \wedge s = d \\ 1, u = s \wedge s \neq d \\ -A^{ud}, u \neq s \wedge s = d \\ 0, 1, u \neq s \wedge s \neq d \end{cases},$$

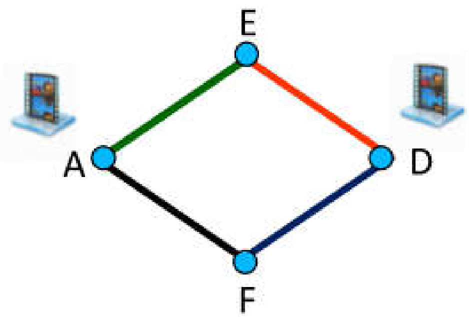
$$\forall ij, kl \in E_P, \forall st, ts \in E_L, \forall u \in N_L, \forall d \in D, \forall s \in N_L$$

Current Issues

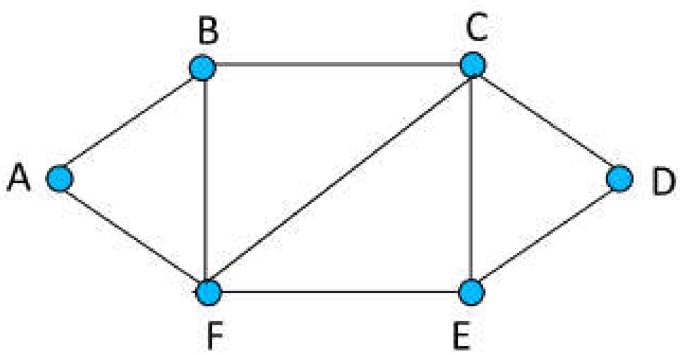
- Multi-dimension variables:
  - F_{ij}^{udst}: 6 dimensions
  - T_{ijkl}^{udst}: 8 dimensions
  - As a result, constraint 9 and constraint 10 is not solvable in Cplex.
- No verification of logical correctness

Optimization Results

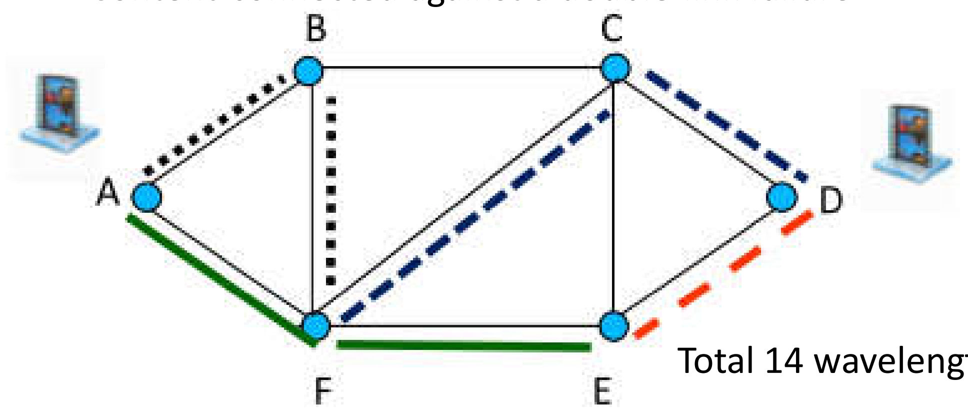
Logical Topology



Physical Topology



Content-connected against a double-link failure



Total 14 wavelength channels

- Mapping:
  - ✓ EA = EF-FA
  - ✓ FA = FB-BA
  - ✓ ED = ED
  - ✓ FD = FC-CD
- Solution is not **optimal**
- A failure on link *FE* and a failure on link *ED* disconnect node **E**
- There is no solution for this problem!!!

IBM CPLEX

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Research Directions (Future Work)

1. Verify the afore-mentioned LP and develop heuristic algorithms
2. There is no guarantee of a feasible solution to content connectivity against a single-link failure and a double-link failure. Feasibility depends on:
   - Physical node degree
   - Logical node degree
   - Number of data centers
   - How networks are connected.

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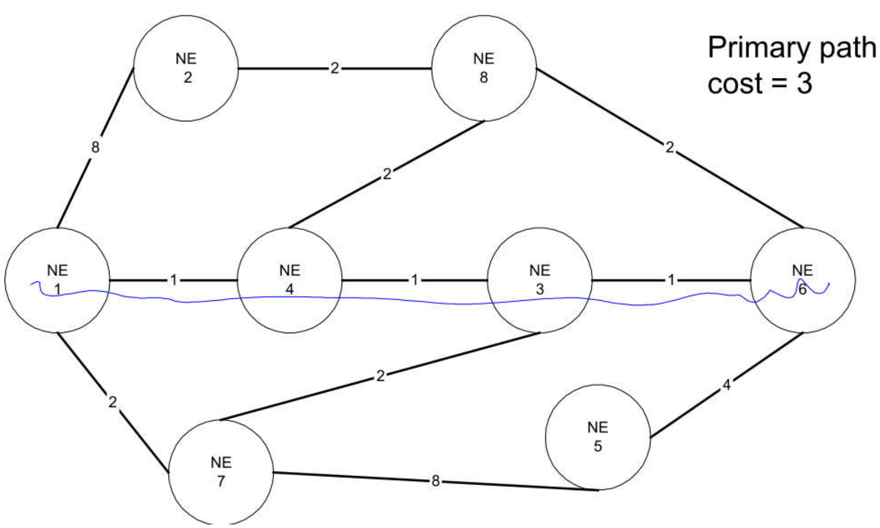
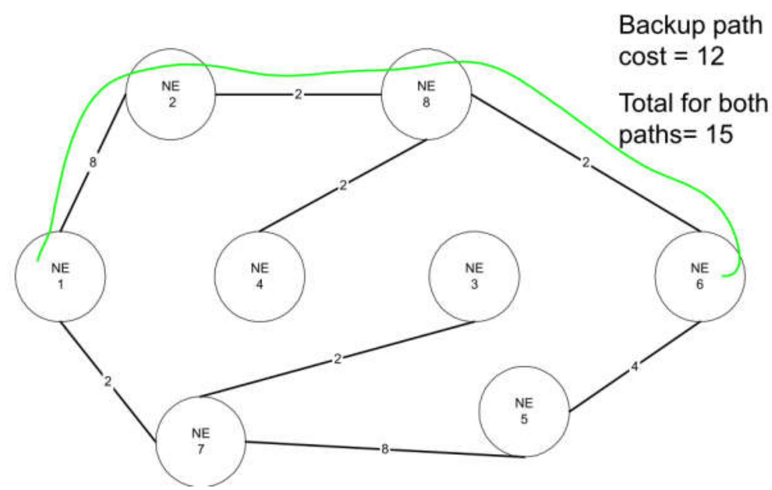
Research Directions (Future Work)

3. Bhandari's Algorithm to find Link-Disjoint Paths





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Research Directions (Future Work)

3. Content Connectivity against a Multi-Link Failure (*n* links)
   - If there are data centers at nodes: no protection required
   - If a node is not data center, remove all combination of *n* links in the physical topology and map logical connections to data centers

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