Networks Lab Meeting

Friday, May 17, 2019

Network Connectivity and Content Connectivity Implementations in IBM Ilog Cplex Opt. Studio

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Outline

1. Survivable mapping in WDM optical networks
2. Implementation of survivable mapping for network connectivity
3. Content connectivity in WDM optical networks

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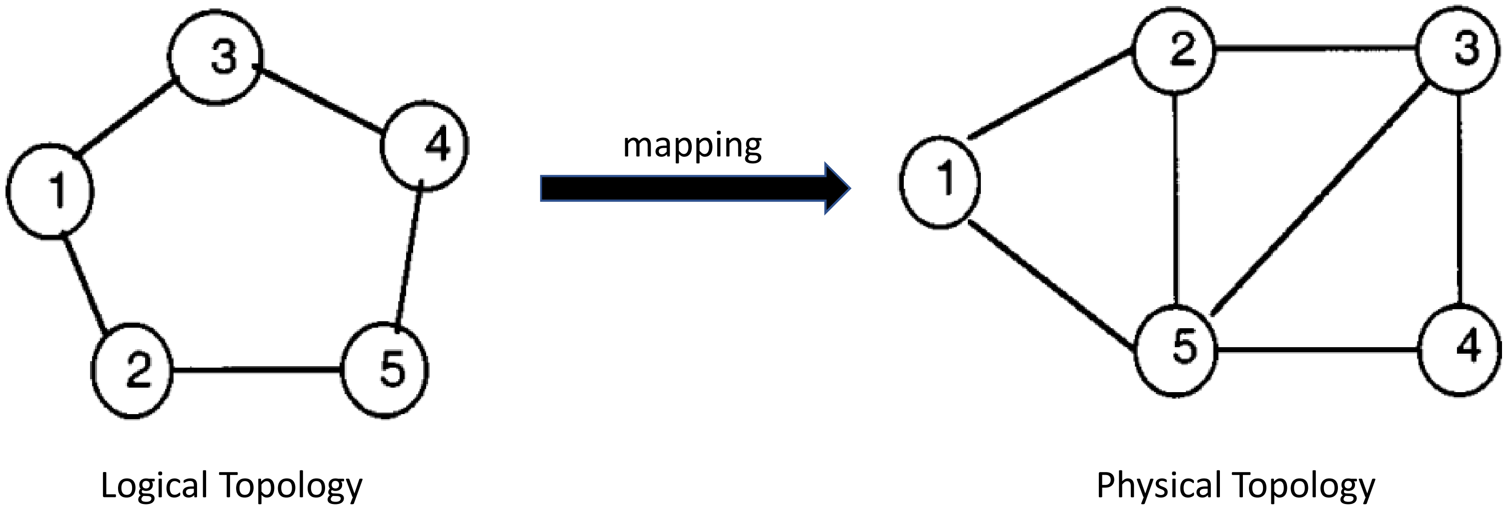
Survivable Mapping in WDM Optical Networks



Survivable mapping: Routing and Wavelength Assignment of the Logical Topology over the Physical Topology such that the Logical Topology is still connected after a certain number of failures

Notation and Assumptions

- ✓ Physical topology *G(N, E)*
- ✓ Logical topology $G(N_L, E_L)$
- ✓ Bidirectional for both physical connection ($(i, j) \in E$ so (j, i)) and logical connection ($(s, t) \in E_L$ so (t, s))
- ✓ With wavelength converters
- ✓ *W* wavelengths per fiber

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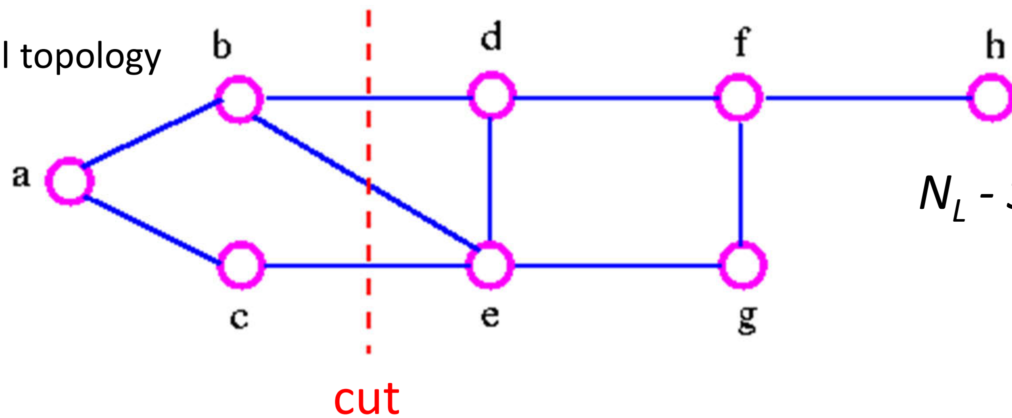
Fundamentals of Survivable Mapping

- *Cut*: Operation dividing all logical nodes (N_L) into 2 parts, S and N_L - S

Logical topology

$S = \{a, b, c\}$



$N_L - S = \{d, e, f, g, h\}$

- *Cutset*: All lightpaths consisting one edges in S and the other in N_L - S

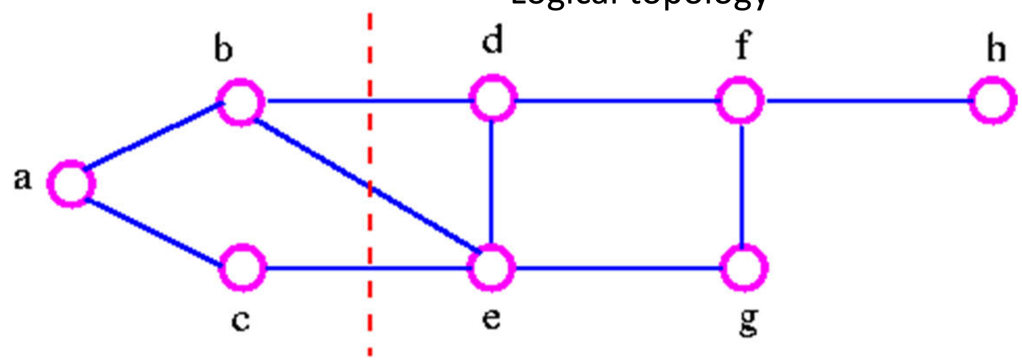
Fundamentals of Survivable Mapping

- $f_{i,j}^{s,t} = 1$ if lightpath from *s* to *t* is mapped over physical link from *i* to *j*
- *E(s, t),* set of all physical links used by lightpath (*s, t*), a logical topology is survivable if and only if for every *cutset*,

$$\bigcap_{(s,t)\in \mathrm{CS}(S, N_L - S)} E(s,t) = \emptyset$$

- In other words, never map all lightpaths (*bd*, *be*, and *ce*) in all cutsets over a single physical link

Logical topology



CUT

ILP Formulation and Implementation

$$\text{Minimize} \sum_{\substack{(i,j)\in E\\(s,t)\in E_L}} f_{ij}^{st}$$

Subject to:

a) Connectivity constraints: for each pair (s,t) in E_L:

$$\sum_{j\ s.t.\ (i,j)\in E} f_{ij}^{st} - \sum_{j\ s.t.\ (j,i)\in E} f_{ji}^{st} = \begin{cases} 1, & \text{if } s=i \\ -1, & \text{if } t=i \\ 0, & \text{otherwise} \end{cases}$$

$\forall i \in N.$

b) Survivability constraints:

$$\begin{matrix}\forall (i,j)\in E \\ \forall S \subset N_L\end{matrix}, \quad \sum_{(s,t)\in \mathrm{CS}(S,N_L-S)} f_{ij}^{st} + f_{ji}^{st} < |\mathrm{CS}(S,N_L-S)|$$

c) Capacity constraints:

$$\forall (i,j)\in E, \quad \sum_{(s,t)\in E_L} f_{ij}^{st} \le W$$

- No wavelength continuity
- Starting point for network connectivity and content connectivity

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ILP Solvers

Approach 1

- Engine: Cplex
- APIs to Java, Python, C++, ...

Approach 2

- Engine: **Cplex**
- Programming Languages used for optimization
- Similar to mathematical expressions
- **OPL** , AMPL...

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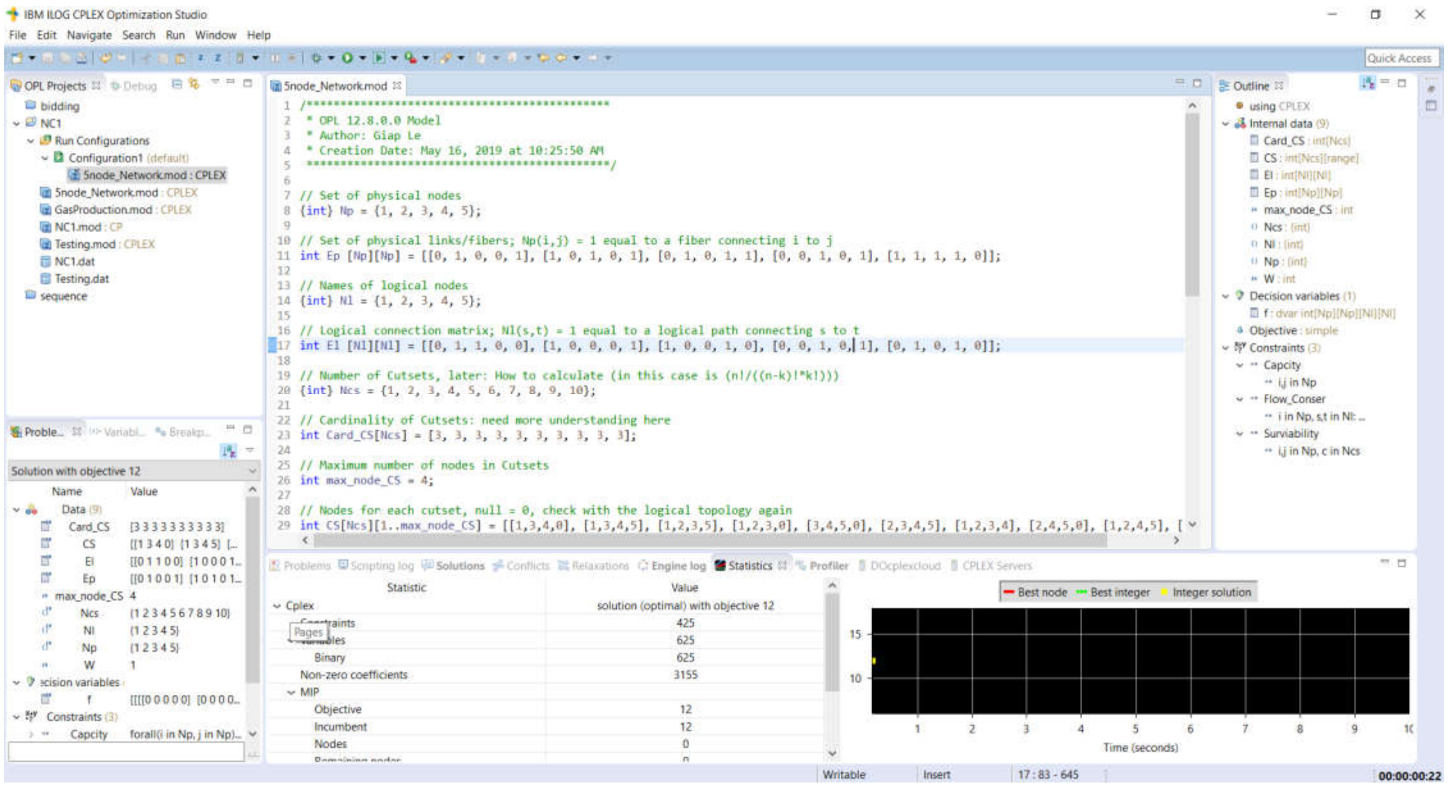
IBM ILOG Optimization Studio



- Licensed for UCD students
- Built for ILPs

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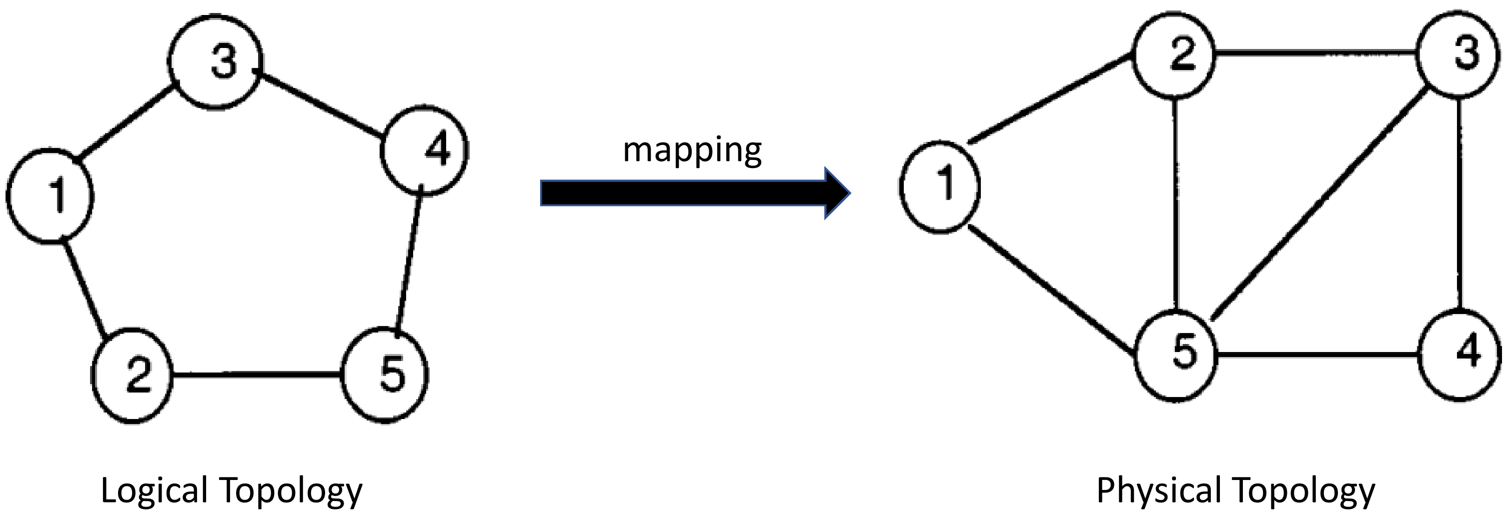
ILP Implementation

- Survivable mapping ILP formulation on slide #8 to following topologies



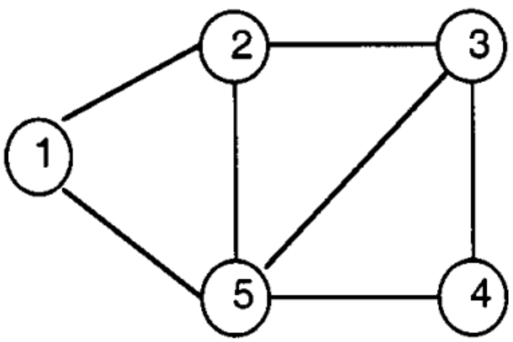
Input Parameters

```
 * OPL 12.8.0.0 Model
 * Author: Giap Le
 * Creation Date: May 16, 2019 at 10:25:50 AM
 *********************************************/

// Set of physical nodes
{int} Np = {1, 2, 3, 4, 5};

// Set of physical links/fibers; Np(i,j) = 1 equal to a fiber connecting i to j
int Ep [Np][Np] = [[0, 1, 0, 0, 1], [1, 0, 1, 0, 1], [0, 1, 0, 1, 1], [0, 0, 1, 0, 1], [1, 1, 1, 1, 0]];

// Names of logical nodes
{int} Nl = {1, 2, 3, 4, 5};

// Logical connection matrix; Nl(s,t) = 1 equal to a logical path connecting s to t
int El [Nl][Nl] = [[0, 1, 1, 0, 0], [1, 0, 0, 0, 1], [1, 0, 0, 1, 0], [0, 0, 1, 0, 1], [0, 1, 0, 1, 0]];

// Number of Cutsets, later: How to calculate (in this case is (n!/((n-k)!*k!)))
{int} Ncs = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};

// Cardinality of Cutsets
int Card_CS[Ncs] = [3, 3, 3, 3, 3, 3, 3, 3, 3, 3];

// Maximum number of nodes in Cutsets
int max_node_CS = 4;

// Nodes for each cutset, null = 0, check with the logical topology again
int CS[Ncs][1..max_node_CS] = [[1,3,4,0], [1,3,4,5], [1,2,3,5], [1,2,3,0], [3,4,5,0], [2,3,4,5], [1,2,3,4], [2,4,5,0], [1,2,4,5], [1,2,5,0]];

int W = 50;  // Number of wavelengths per fiber, assuming one single fiber per direction

dvar int f[Np][Np][Nl][Nl] in 0..1;   // All possible conbinations, total variables: Np^2 * Nl^2 = 625 for this network
```

decision variables

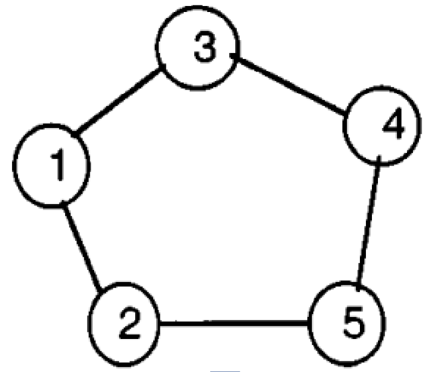
Topology Modelling

Physical topology



Np (size 5)	Np (size 5)				
	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	0	1
3	0	1	0	1	1
4	0	0	1	0	1
5	1	1	1	1	0

Logical topology



Nl (size 5)	Nl (size 5)				
	1	2	3	4	5
1	0	1	1	0	0
2	1	0	0	0	1
3	1	0	0	1	0
4	0	0	1	0	1
5	0	1	0	1	0

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ILP Formulation

```
minimize sum(i,j in Np: Ep[i][j] == 1, s,t in Nl: El[s][t] == 1) f[i][j][s][t];  // Calculating for valid combinations only, El[s][t] == 1

subject to {

    Flow_Conser: forall (i in Np, s,t in Nl: El[s][t] == 1) {

        if (i == s) {
        sum (j in Np: Ep[i][j] == 1) (f[j][i][s][t] - f[i][j][s][t]) == -1;
        }

        if (i == t) {
        sum (j in Np: Ep[i][j] == 1) (f[j][i][s][t] - f[i][j][s][t]) == 1;
        }

        if ((i != s) && (i != t)) {
        sum (j in Np: Ep[i][j] == 1) (f[j][i][s][t] - f[i][j][s][t]) == 0;
        }
      }

    Capcity: forall (i,j in Np)
      sum (s,t in Nl) f[i][j][s][t] <= W;

    Surviability: forall (i,j in Np, c in Ncs) {

      sum (s,t in {CS[c][1], CS[c][2], CS[c][3], CS[c][4]}: El[s][t] == 1) (f[i][j][s][t] + f[j][i][s][t]) <= Card_CS[c] - 1;

    }
}
```

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Results

```
// solution (optimal) with objective 12
// Quality Incumbent solution:
// MILP objective                                1.2000000000e+01
// MILP solution norm |x| (Total, Max)           1.20000e+01  1.00000e+00
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
```

Total 12 Wavelength Channels

i	j	s	t	$f_{i,j}^{s,t}$
Np (size 5)	Np (size 5)	Nl (size 5)	Nl (size 5)	Value
5	4	5	4	1
5	3	1	3	1
5	2	5	2	1
5	1	3	1	1
4	5	4	5	1
4	3	4	3	1
3	5	3	1	1
3	4	3	4	1
2	5	2	5	1
2	1	2	1	1
1	5	1	3	1
1	2	1	2	1
5	5	5	5	0
5	5	5	4	0
5	5	5	3	0
5	5	5	2	0

Notes:
- *p*: physical nodes
- *l*: logical nodes
- Bidirectional connections

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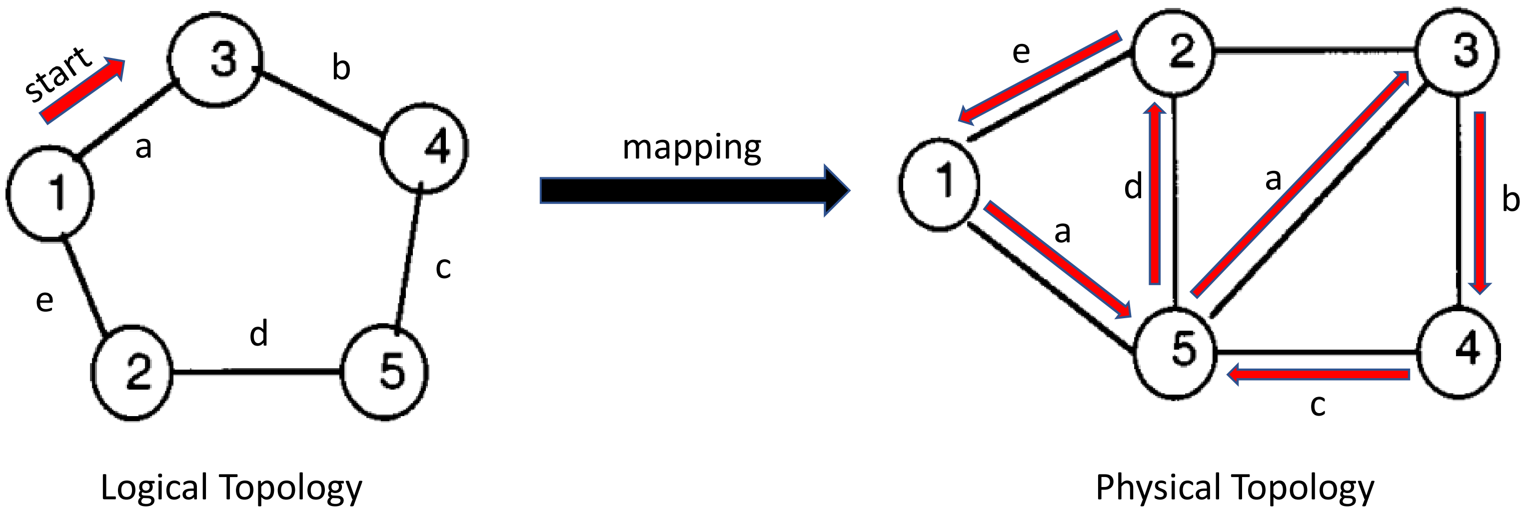
Visual Mapping



- Bidirectional: total 12 Wavelength channels
- Survivable for single-link failures
- Wavelength colors should avoid conflicts

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Future Work

- Formulate and implement network and content connectivity ILPs
- Analyze the cost of protection against various failure scenarios
- Develop heuristic algorithms for selected ILPs
- Important dates: June 01, 2019 (JUNO2 meeting schedule), OFC 2020